ASME International

# AIAA 97-0939

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**35th Aerospace Sciences Meeting & Exhibit**
January 6-10, 1997 / Reno, NV

AIAA-97-0939

# NUMERICAL TECHNIQUES FOR THE IMPROVED PERFORMANCE OF A FINITE ELEMENT APPROACH TO WIND TURBINE AEROELASTICS

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## ABSTRACT

A numerical technique has been developed which allows the use of finite element techniques to be used for the modelling of a coupled rotor tower structure. The method is based upon the fact that the equations of motions of periodic and that the number of terms which are time dependent are small in comparison to the number of degrees of freedom. Using a wind turbine as an example this approach allows the equations of motion to be solved in real time in real time and therefore enabling the finite element approach to be readily used in the design phase.

## INTRODUCTION

It is generally accepted that the finite element approach to the modelling of complex structures will yield the most accurate results. However, for wind turbines where the response is required to a combination of deterministic and stochastic inputs the duration of the simulation period often prohibits this approach and the designer is required to find an alternative approach which more often than not will include a number of simplifying assumptions. The purpose of this paper is to introduce a numerical technique which will allow a conventional finite element method to be used in a parametric approach to the design of and analysis of measured data from a horizontal axis wind turbine.

The paper concentrates on the numerical technique for improving the speed of calculation and only mentions the loading in passing as this is well documented in other papers on the subject [2].

## EQUATIONS OF MOTION

The formulation of the equations of motion for the wind turbine relies heavily upon the work of Lobitz [2] and [3] at Sandia Laboratories. The route pioneered by Lobitz, starts with the formation of the mass, damping and stiffness matrices of the rotor and tower as two separate entities. The two models are then coupled together through the use of a time dependent connection matrix.

The tower and rotor can be considered as separate sub-structures represented by the following equations of motion:

$$[M_T]\{\ddot{U}_T\}+[C_T]\{\dot{U}_T\}+[K_T]\{U_T\}=\{F_T\} \quad (1)$$

$$[M_R]\{\ddot{U}_R\}+[C_R+C_\Omega]\{\dot{U}_R\}+[K_R-S_\Omega]\{U_R\} = \{F_R\} \quad (2)$$

Here the subscripts refer to the tower and rotor systems respectively. The quantities $C_\Omega$ and $S_\Omega$, which are derived from rotating co-ordinate effects, are the Coriolis and centrifugal softening matrices respectively. The stiffness matrix, $K_R$ is complicated by the fact that it contains additional terms due to centrifugal stiffening. The vectors on the right hand side of the equations represent the applied forces (aerodynamic and gravity) and in general are functions of the displacements {U} and their derivatives.

Equations (1) and (2) can be combined into a single matrix equation as follows:

$$\begin{bmatrix} M_T & 0 \\ 0 & M_R \end{bmatrix}\begin{Bmatrix} \ddot{U}_T \\ \ddot{U}_R \end{Bmatrix}+\begin{bmatrix} C_T & 0 \\ 0 & C_R+C_\Omega \end{bmatrix}\begin{Bmatrix} \dot{U}_T \\ \dot{U}_R \end{Bmatrix} +\begin{bmatrix} K_T & 0 \\ 0 & K_R-S_\Omega \end{bmatrix}\begin{Bmatrix} U_T \\ U_R \end{Bmatrix}=\begin{Bmatrix} F_T \\ F_R \end{Bmatrix} \quad (3)$$

Denoting the time-dependent constraint relation which connects the rotor to the tower as [T] then it is convenient to use a single vector {U} to describe the displacements of the system as a whole, e.g.

$$\begin{Bmatrix} U_T \\ U_R \end{Bmatrix}=[T]\{U\} \quad (4)$$

Similar relationships can be derived for the velocities and accelerations:

$$\begin{Bmatrix} \dot{U}_T \\ \dot{U}_R \end{Bmatrix}=[\dot{T}]\{U\}+[T]\{\dot{U}\} \quad (5)$$

$$\begin{Bmatrix} \ddot{U}_T \\ \ddot{U}_R \end{Bmatrix}=[\ddot{T}]\{U\}+2[\dot{T}]\{\dot{U}\}+[T]\{\ddot{U}\} \quad (6)$$

Substituting equations (4), (5) and (6) into (3) and using [M], [C],[K] and {F} to represent the whole system yields:

$$\begin{aligned}&([T]^T[M][T])\{\ddot{U}\}+([T]^T[C][T]\\&+2[T]^T[\dot{T}])\{\dot{U}\}\\&+([T]^T[K][T]+[T]^T[M][\ddot{T}]\\&+[T]^T[C][\dot{T}])\{U\}=[T]^T\{F\}\end{aligned} \qquad (7)$$

The vector {F} will in general be a function of time and of the vector {U} and its derivatives.

Through the introduction of the time dependent connection matrix [T] the solution of this equation can be time consuming unless a number of simplifications are made

## SIMPLIFICATIONS

As pointed out by Lobitz [3] the connecting matrix, [T] only modifies terms in the matrices associated with the tower or connection nodes, and by a judicious selection of the physical modelling at these points certain terms in equation (7) can be simplified. For example, if the tower connection node possesses only lumped translational mass then the terms

$$[T]^T[M][T],\ [T]^T[M][\dot{T}] \quad and\ [T]^T[M][\ddot{T}]$$

are time invariant and only need to be calculated once. Additionally if the tower connection node is not directly involved in any damping then also the term

$$[T]^T[C][T]$$

becomes time invariant and the term

$$[T]^T[C][\dot{T}]$$

and vanishes. Equation (7) hence simplifies to:

$$\begin{aligned}&[M_o]\{\ddot{U}\}+[C_o]\{\dot{U}\}+([K_o]\\&+[T]^T[K][T])\{U\}=[T]^T\{F\}\end{aligned} \qquad (8)$$

and therefore the only term which requires to be calculated at each time step is

$$[T]^T[K][T]$$

## INTEGRATION OF THE EQUATIONS OF MOTION

The loading vector {F} will comprise deterministic (wind shear, tower shadow, gravity etc.) and stochastic (turbulence) components and therefore to obtain a statistically meaningful result a long integration time is required. Unless an integration scheme is used which is numerically fast and accurate then having a system with so many degrees of freedom would not be a tractable approach.

For equations of motion with constant coefficients implicit methods are unconditionally stable which allows for the arbitrary selection of the time step required to fully model the highest frequency of interest. However, the equations motion here unfortunately contain coefficients which are time dependent and therefore unconditional stability can not always be guaranteed. An alternative approach would be through the use of a explicit integration scheme such as Runge-Kutta or a predictor corrector technique based on Gear's backward difference method. As the equations of motion are likely to be *stiff* then the time step required to maintain stability would be prohibitively short to allow a stochastic response to be analysed.

The technique which has been adopted here is based on the Newmark-Beta implicit integration scheme [4]:

$${}^{t+\Delta t}\dot{U}={}^{t}\dot{U}+\left((1-\delta){}^{t}\ddot{U}+\delta\,{}^{t+\Delta t}\ddot{U}\right)\Delta t \qquad (9)$$

$${}^{t+\Delta t}U={}^{t}U+{}^{t}\dot{U}\Delta t+\left((1/2-\alpha){}^{t}\ddot{U}+\alpha\,{}^{t+\Delta t}\ddot{U}\right)\Delta t^2 \qquad (10)$$

To avoid confusion and for the sake of brevity the symbols of [] and {} representing matrices and vectors have been dropped.

In addition to equations (9) and (10) the equilibrium equation at time t+ $\Delta$ t also requires solution:

$$M\,{}^{t+\Delta t}\ddot{U}+C\,{}^{t+\Delta t}\dot{U}+K\,{}^{t+\Delta t}U={}^{t+\Delta t}R \qquad (11)$$

The parameters $\alpha$ and $\delta$ can be determined to obtain integration accuracy and stability but are usually selected according to the equation:

$$\delta\geq 0.5;\quad \alpha\geq 0.25(0.5+\delta)^2 \qquad (12)$$

The following additional integration constants are also required:

$$\begin{aligned}&a_0=\frac{1}{\alpha\Delta t^2};\quad a_1=\frac{\delta}{\alpha\Delta t^2};\\&a_2=\frac{1}{\alpha\Delta t};\quad a_3=\frac{1}{2\alpha}-1;\\&a_4=\frac{\delta}{\alpha}-1\\&a_5=\frac{\Delta t}{2}\left(\frac{\delta}{\alpha}-2\right);\quad a_6=\Delta t(1-\delta);\\&a_7=\delta\Delta t\end{aligned} \qquad (13)$$

At each time step the effective stiffness matrix and load is required:

$$^{t+\Delta t}K_e = {}^{t+\Delta t}K + a_0\, {}^{t+\Delta t}M + a_1\, {}^{t+\Delta t}C \quad (14)$$

$$\begin{aligned} {}^{t+\Delta t}R_e = {}^{t+\Delta t}R + {}^{t+\Delta t}M\left(a_0\,{}^tU + a_2\,{}^t\dot{U} + a_3\,{}^t\ddot{U}\right) \\ + {}^{t+\Delta t}C\left(a_4\,{}^tU + a_5\,{}^t\dot{U} + a_6\,{}^t\ddot{U}\right) \end{aligned} \quad (15)$$

From which the displacements can be calculated:

$$^{t+\Delta t}K_e\, {}^{t+\Delta t}U = {}^{t+\Delta t}R_e \quad (16)$$

and the accelerations and velocities:

$$^{t+\Delta t}\ddot{U} = a_0\left({}^{t+\Delta t}U - {}^tU\right) - a_2\,{}^t\dot{U} - a_3\,{}^t\ddot{U} \quad (17)$$

$$^{t+\Delta t}\dot{U} = {}^t\dot{U} + a_6\,{}^t\ddot{U} + a_7\,{}^{t+\Delta t}\ddot{U} \quad (18)$$

Even though the application of this integration scheme allows for the possibility of selecting large integration steps the time dependent coefficients require that equation (16) be solved at each time step. If the system had not contained time dependent coefficients then it would have been possible to use LU decomposition with pivoting and back-substitution.

To avoid this problem recourse can be made to the fact that the effective stiffness matrix is *sparse* and only a small number of the matrix elements are varying with time. In this case the Woodbury formula [5] which is a block-matrix version of the Sherman-Morison formula can be used.

Suppose that it is possible to write the matrix to be inverted as:

$$[A] \rightarrow [A] + \{w\} \otimes \{v\} \quad (19)$$

for some vectors {w} and {v}. If {w} is a unit vector $\{e_i\}$ then equation (19) adds the components of v to the ith row. Similarly if {v} is a unit vector $\{e_j\}$then equation (19) adds the components of {w} to the jth column. If both {w} and {v} are proportional to unit vectors $\{e_i\}$ and $\{e_i\}$ respectively, then a term is added only to element $a_{ij}$.

The Sherman-Morrison formula gives the inverse as follows:

$$\begin{aligned} ([A] + \{w\} \otimes \{v\})^{-1} = [A]^{-1} \\ - \frac{\left([A]^{-1}\{w\}\right) \otimes \left(\{v\}[A]^{-1}\right)}{(1+\lambda)} \end{aligned} \quad (20)$$

where [A] is a N by N matrix and

$$\lambda \equiv \{v\}[A]^{-1}\{w\} \quad (21)$$

The whole procedure requires only $3N^2$ multiplies whereas standard methods require of the order of $N^3$ multiplies, a saving of a factor of N. The Woodbury formula extends the above to allow more than a single correction term, viz.

$$\begin{aligned} \left([A] + [W][V]^T\right)^{-1} = [A]^{-1} \\ -\left([A]^{-1}[W]\left([I]+[V]^T[A]^{-1}[W]\right)^{-1}[V]^T[A]^{-1}\right) \end{aligned} \quad (22)$$

where [W] and [V] are N by P matrices, with P<N. P is the number of correction terms. More often than not $[A]^{-1}$ is not explicitly kept or obtained and therefore we may use equation (22) in the following manner:

$$\left([A] + \sum_{k=1}^{p} \{w_k\} \otimes \{v_k\}\right)\{x\} = \{b\} \quad (23)$$

First solve the P auxiliary equations, noting that the each vector {w} contains only unity at the locations to be changed in the row in the kth column

$$\begin{aligned} &[A]\{z_1\} = \{w_1\} \\ &[A]\{z_2\} = \{w_2\} \\ &\ldots \\ &[A]\{z_k\} = \{w_k\} \end{aligned} \quad (24)$$

and construct the matrix [Z] by columns from the z's obtained,

$$[Z] \equiv \{z_1\} \ldots \{z_P\} \quad (25)$$

Next, do the P by P matrix inversion

$$[H] \equiv \left([I] + [V]^T[Z]\right)^{-1} \quad (26)$$

Finally solve one further auxiliary problem

$$[A]\{y\} = \{b\} \quad (27)$$

In terms of the above quantities, the solution is given by

$$\{x\} = \{y\} - [Z]\left([H]\left([V]^T\{y\}\right)\right) \quad (28)$$

As noted above this procedure will result in an increase in computational speed by a factor of N. Typically for a horizontal axis wind turbine N will in the region of 100 to 200 depending upon its complexity.

## LIMITATIONS

The model as currently formulated has a number of possible limitations. The first stems from the assumption that the two sub-structures can be coupled together through a time dependent constraint relationship as represented by equation (4) and as such the origin of both co-ordinate systems are fixed. All deformations within the rotating system will be relative to an origin fixed at the junction between the two sub-structures. For a turbine which has a