# A Review of MCP Techniques

Report No: 01327R00022

Issue No: 03

Date: 1 December 2004

# A Review of MCP Techniques

| | |
|---|---|
| Report No: | 01327R00022 |
| Issue No: | 03 |
| Classification: | COMMERCIAL IN CONFIDENCE |

| | | | |
|---|---|---|---|
| Prepared: | **Mike Anderson** | Signed Electronically | 1 December 2004 |
| Checked: | **Jeremy Bass** | Signed Electronically | 1 December 2004 |
| Approved: | **Mike Anderson** | Signed Electronically | 1 December 2004 |



Signed, Checked and Approved Electronically 1 December 2004

# Revision History

| Issue | Date | Nature And Location Of Change |
|---|---|---|
| 01 | 18/06/04 | First Created |
| 02 | 26/07/04 | Additional results included |
| 03 | 01/12/04 | Typographical errors corrected. |

CONTENTS

## 1.0 INTRODUCTION

This report reviews a range of methods used to estimate the long term wind speed distribution at potential wind farm sites, where only a limited amount of on-site data are available.

The methods which are reviewed here comprise a family of generic ‘measure-correlate-predict’ (MCP) methodologies. These methods proceed by ‘measuring’ the winds at a target site, ‘correlating’ them with winds from a nearby reference site, and then by applying these correlations to historical data from the reference site, to ‘predict’ the long term wind resource of the target site.

## 2.0 OUTLINE OF MCP METHODOLOGY

The general methodology of the MCP process proceeds as follows:

i) collect wind data at the predictor site for an extended period;

ii) identify a reference site, for which high quality, long term records exist, in the vicinity of the predictor site, and which has a similar exposure - this is hereafter referred to as the ‘reference’ site;

iii) obtain wind data from the reference site for the same time period as for the predictor site - this period is hereafter referred to as the ‘concurrent period’.

iv) establish a relationship between the data from the reference and predictor sites for the concurrent period;

v) obtain wind data from the reference site for a historic period of 10 to 20 years duration - this period is hereafter referred to as the ‘historic’ period;

vi) apply the relationship determined from (iv) above to the historic data from the reference site to ‘predict’ what the winds would have been at the predictor site over that period. Note that this is a prediction of the winds that would have been observed had measurements been made at the predictor site for the same period as the historic data, rather than a prediction of winds that will be observed in future.

These steps are illustrated in Fig 2.1.

The key to any MCP technique is the algorithm used in (iv). Most MCP techniques use direction sectored regression analysis to establish a relationship between wind speed and direction at the reference site and the wind speed at the potential wind farm site. The procedure is described below:

i) sector the concurrent data into bins based on the wind direction measured at the reference site.

ii) within each sector a regression analysis is carried out to establish the correlation between reference wind speed and potential wind farm site wind speed.

Thus, by taking the wind speed and direction at the reference station from the historical data, the required sector is obtained and the functional relationship applied to calculate the wind speed at the potential wind farm site. For the rest of this report the potential wind farm site will be referred to as the ‘predictor’.

## 3.0 DATA SETS

The data used in this analysis is based on that used by Bass [2000], but modified as follows:

- removal of sites where an MCP approach is clearly not appropriate (very low correlation);
- removal of sites where there is less than 6 months of data;
- additional sites added.

In addition individual records within a dataset were removed when invalid data was present for either wind speed or direction data. Also calms (wind speed < 0.1 m/s) were also removed. The total data base comprises 53 unique sites containing a total of 110 years of concurrent hourly average data. The distribution of mean wind speeds and data lengths in the database is shown in Figures 3.1 and 3.2. The distribution of the overall product-moment correlation coefficient, *r*, is presented in Table 3.1. It should be noted that these values represent the overall coefficient and that the coefficient within a given sector will be greater than these values.

| Correlation Coefficient (*r*) | Number of Datasets | Description |
|---|---|---|
| 0.5 to 0.6 | 2 | Very poor |
| 0.6 to 0.7 | 12 | Poor |
| 0.7 to 0.8 | 35 | Moderate |
| 0.8 to 0.9 | 37 | Good |
| 0.9 to 1.0 | 20 | Very good |

Table 3.1 – Correlation Coefficients

In the original study [Bass, 2000] the two halves of the data were taken as consecutive halves, i.e. if the total length of data from the ‘reference-predictor’ pair was 24 months then the first half would be the first 12 months and second half the next 12 months. In this study an additional data set was formed by taking alternate samples, i.e. if the total length of data from the ‘reference-predictor’ pair was 24 months comprising N records then the first half would be made up from the 1,3,5…N-1 (odd) sample and the second half from the 2,4,6….N (even) samples. Within this report the original method for data preparation will be known as “sliced” and the alternative as “diced”. By introducing a “diced” data set and comparing the results to the “sliced” data set, this will allow the actual performance of each of the methods to be established and not masked by trends in the time histories. It should be noted that the “diced” data set is not an alternative to the “sliced” data set for real world applications.

Both methods result in the ‘reference-predictor’ pair being split into two halves, allowing two separate sets of predictions to be made that can then be compared with the real, measured data. This gives a total of 106 data sets of hourly averaged wind data that can be used for MCP performance assessment. The two halves of the data are used as follows:

- the first half of the data from both reference and predictor sites is regarded as ‘concurrent data’ and used to generate a relationship between winds at the two sites. The second half of the reference site data is regarded as ‘historic’ data and, by applying the relationship obtained from the first half, used to generate MCP predictions that can be directly compared with the measured, predictor site data.

- the second half of the data from both reference and predictor sites is regarded as ‘concurrent data’ and used to generate a relationship between winds at the two sites. The first half of the reference site data is regarded as ‘historic’ data and, by applying the relationship obtained from the second half, used to generate MCP predictions that can be directly compared with the measured, predictor site data.

## 4.0 METHODS REVIEWED

The models which are reviewed fall into two classes, linear and non-linear and can be described mathematically:

$$\begin{aligned} V_{c,pred} &= f(V_{c,ref}, \theta_{c,ref}) \\ \theta_{c,pred} &= g(V_{c,ref}, \theta_{c,ref}) \end{aligned} \qquad [1]$$

where $f, g$ represent the functional relationships between the ‘concurrent data’ for the two sites. Subscripts denote the data set; concurrent $c$, historic $h$, reference $ref$ or predictor $pred$.

Techniques exists [Bass, 1999] for including wind veer into the MCP process but for the purposes of this review it will be assumed, without loss of applicability, that the wind direction at the reference and predictor sites are related as:

$$\theta_{pred} = \theta_{ref} \qquad [2]$$

### *4.1 Linear Model*

#### 4.1.1 Ratio of Means

a) Single Sector

The wind speed at the predictor site is given by:

$$\begin{aligned} V_{h,pred} &= f(V_{h,ref}) \\ f(V_{h,ref}) &= \frac{\overline{V}_{c,pred}}{\overline{V}_{c,ref}} V_{h,ref} \end{aligned} \qquad [3]$$

The over bar refers to average of the concurrent data.

b) Multiple Sectors

For the $i^{th}$ sector (normally twelve; 30 degree sectors)

$$V^i{}_{h,pred} = f^i(V_{h,ref}) \\ f^i(V_{h,ref}) = \frac{\overline{V}^i{}_{c,pred}}{\overline{V}^i{}_{c,ref}} V^i{}_{h,ref} \quad [4]$$

The superscript denotes the wind direction sector.

### 4.1.2 Least Squares

a) $y = \beta x + \alpha$ "LSQ2"

For the $i^{th}$ sector (normally twelve) the gradient and the offset are derived by performing a standard least squares regression.

$$V^i{}_{h,pred} = f^i(V_{h,ref}) \\ f^i(V_{h,ref}) = \beta^i V_{h,ref} + \alpha^i \quad [5]$$

b) $y = \beta x$ "LSQ1"

For the $i^{th}$ sector (normally twelve) the gradient is derived by performing a standard least squares regression.

$$V^i{}_{h,pred} = f^i(V_{h,ref}) \\ f^i(V_{h,ref}) = \beta^i V_{h,ref} \quad [6]$$

### 4.1.3 Orthogonal Regression

The method of orthogonal regression has a long and distinguished history in statistics and economics. It has been viewed as superior to ordinary least squares in certain situations. Orthogonal regression is one of the standard linear regression methods to correct for the effects of measurement and equation error. Details of this technique can be found in Annex A. This form of regression has been applied in the following ways:

a) $y = \beta x + \alpha$ "OR2"

For the $i^{th}$ sector the gradient is derived by performing a standard orthogonal regression as detailed in Annex A;

$$V^i{}_{h,pred} = f^i(V_{h,ref}, \lambda) \\ f^i(V_{h,ref}, \lambda) = \beta^i(\lambda) V_{h,ref} + \alpha^i(\lambda) \quad [7]$$

The value $\lambda$ takes on is somewhat subjective and will be discussed in a later section.

b $y = \beta x$ "OR1"

For the $i^{th}$ sector the gradient is derived by performing a standard orthogonal regression as detailed in Annex A;

$$V^i_{h,pred} = f^i(V_{h,ref}) \qquad [8]$$
$$f^i(V_{h,ref}) = \beta^i V_{h,ref}$$

c) $y = \beta x + \alpha$ "Method A"

For the $i^{th}$ sector (normally twelve) the gradient and the offset are derived by performing a modified orthogonal regression as detailed in Annex A3;

$$V^i_{h,pred} = f^i(V_{h,ref}) \qquad [9]$$
$$f^i(V_{h,ref}) = \beta^i V_{h,ref} + \alpha^i$$

d) $y = \beta x$ "Method B"

For the $i^{th}$ sector (normally twelve) the gradient is derived by performing a standard orthogonal regression as detailed in Annex A1;

$$V^i_{h,pred} = f^i(V_{h,ref}) \qquad [10]$$
$$f^i(V_{h,ref}) = \beta^i V_{h,ref}$$

However, prior to performing the regression analysis the data in each sector is censored to remove all data below 3 m/s at the reference site and approximately $3\beta^i$ m/s at the predictor site [Garrad Hassan, 2003]

### *4.2 Discussion*

It is clear from the above that when a two parameter fit is used it will introduce an offset into predicted wind speed distribution. In other words calms or periods of very low wind speeds will never occur. This is physically unrealistic, but the extent to which it impacts upon the predicted wind speed or the derived energy yield is considered later in the report.

As two parameter fits (orthogonal and least squares) essentially take moments about the mean of the data these parameters will therefore be as equally sensitive to high wind speed as they are to low wind speeds. One of the underlying premises of an MCP analysis is that potential flow exists at both sites and therefore a linear relationship will exist. If potential flow does exist, then it is more likely to occur at higher wind speeds. For this reason the removal of low wind speed data has been suggested coupled with the use of a single parameter fit [Garrad Hassan, 2003]. Where the climatology, mean wind speed and distribution, of the two sites are similar then this route may be appropriate. However, in most cases the mean wind speed at the two sites is very different and the premise that potential flow exists simultaneously at both sites will certainly not occur and certainly not at low wind speeds. If a single parameter fit is used this would imply that when low speeds occur at the reference the same will occur at the predictor site. This certainly does not exist for a considerable number of sites, see Figure 4.1.

## *4.3 Non Linear Models*

Rebbeck [1996] reviewed a number of non-linear models ranging from higher order polynomials through cubic splines to complex surface fitting. He concluded that none of these models performed substantially better than a conventional two parameter least squares linear model. Bass [2000], also came to the same conclusion after investigating the use of neural networks using various network architectures, including: Multi-Layer Perceptrons (MLP); Radial Basis Function Networks (RBF) & Generalised Regression Neural Networks (GRNN).

### 4.3.1 Non Linear Mapping

Instead of trying to prescribe a linear relationship between two variables this technique uses their joint probability distribution. Mortimer [1994] proposed a similar model but instead of allowing the measured data to generate the joint probability distribution he used a triangular distribution having a prescribed mean and width (standard deviation). Allowing the joint probability distribution to be determined by the measured data removes the restriction of Mortimer as well as removing the assumptions from the linear models that the residuals have to be normally distributed.

For a given wind direction sector a scatter plot of the two sets of data can be formed, from which a joint probability distribution constructed, see Figures 4.2 and 4.3. For a given reference wind speed there is no longer one unique value of the site wind speed, but a range, see Figure 4.4 (equivalent to a vertical slice through the joint probability distribution).

Assuming that the joint probability distribution of concurrent wind speeds, for the $i^{th}$ sector can be represented by $p^i(V_{ref}, V_{pred})$ then the predicted historical wind speed distribution is given by

$$K^i(V_{pred}) = \int_0^\infty p^i(V_{ref}, V_{pred}) J^i(V_{ref}) N^i(V_{ref}) dV_{ref} \qquad [11]$$

where

$$N^i(V_{ref}) = \frac{1}{\int_0^\infty p^i(V_{ref}, V_{pred}) dV_{pred}} \qquad [12]$$

is a normalisation factor and $J^i(V_{ref})$ is the historical distribution of wind speeds. As the data is not continuous, but discrete, a matrix formulation of the above is easier to work with. For this reason the technique is known as the “Matrix” method.

If the joint probability distribution of concurrent wind speeds is fully populated, for all wind speeds and directions for which historical data exists then, no further assumptions have to be made in the application of this method. Unfortunately, this condition is unlikely to occur in practice and therefore where gaps occur or high wind speed data does not exist then a method for synthesizing the missing data is required. To ensure that the method is robust a gap is defined as where there is less than 10 hours of data. The approach which has been adopted is based on fitting a two parameter least squares straight line through the concurrent data. For each vertical slice, for which data does not exist, a distribution of wind speeds is generated having a mean based on the least squares fit and a standard deviation of 15% of the mean.

## 5.0 ANALYSIS AND RESULTS

### *5.1 Orthogonal Regression – Determination of $\lambda$*

As discussed in Annex A the error in variables orthogonal regression analysis requires knowledge of $\lambda$ . If we neglect the error in the equation then from equation [A.13] we find $\lambda \approx 1$. If we include the error in the equation then we have $\lambda \geq 1$ (see Annex A). Through trial and error a relationship was derived which minimised the bias error in the mean wind speed (see section 5.2.1). The derived empirical relationship is

$$\lambda = 16\sqrt{\left(\frac{1}{|r|}\frac{\overline{V}_{c,ref}}{\overline{V}_{c,pred}}\right)} \qquad [13]$$

where, $r$ , is the product- moment correlation coefficient.

### *5.2 All Data*

#### 5.2.1 Back Predicted Mean Wind Speed

Once the relationship between the concurrent data sets has been established it is then applied to concurrent reference data set and the results compared with the concurrent predictor data. For all two parameter regression equations and the Matrix method the results are trivial in that there is a unity relationship. For all other methods which do not fall into this category this will not hold. Shown in Figure 5.1 are results of performing this analysis using Method B. It is clear from this that the method is biased in that it underestimates the back predicted wind speed by 3%. The reason for this is thought to be a combination of forcing the regression line through the origin and the censoring of low wind speed data (see section 5.2.2 (b)). Results for other single parameter fits are presented in Figure 5.2 and 5.3 and these show similar results.

#### 5.2.2 Predicted Historical Mean Wind Speed (Bias Error)

The normalised bias error is defined as:

$$\mathrm{E}_V = \frac{\overline{V}_{h,pred} - \overline{V}_{h,act}}{\overline{V}_{h,act}} \qquad [14]$$

where $\overline{V}_{h,pred}$ and $\overline{V}_{h,act}$ are the predicted and actual mean wind speeds for the historical period.

The error in an MCP analysis comprises a number of elements:

- suitability of the applied regression analysis;
- errors in the fitting of the data;
- instrumentation accuracy;
- statistical differences between concurrent and historical periods.

The analysis undertaken here only addresses the first two and therefore any implied errors which this analysis suggests are likely to be a minimum.

a) Sliced Data

Presented in Figure 5.4(a) is the bias error, averaged over all 106 “sliced” data sets. With the exception of LSQ1 all the methods have a bias error of less than 2%. Interestingly, OR(1) has a positive bias error whereas for Method B it is negative even though they are essentially the same method. The difference can probably be ascribed to the data censoring in Method B. The Matrix method has the smallest bias. The bias standard deviations $\sigma_{error}$, presented in Figure 5.4(b), are similar for all methods and this probably reflect the fact that the underlying error in any MCP technique is predominately due to the concurrent period not being representative of the historical period. Assuming that the data sets are independent then the bias error is given by

$$\mathrm{E}_{\sigma} = \frac{\sigma_{error}}{\sqrt{N-1}} \qquad [15]$$

Taking $N$=106 and from Figure 5.4(b) $\sigma_{error}$ = 5% we have from the above equation that the bias error is approximately 1%. From Figure 5.4(a) it can be seen that, with the exception of the LSQ1 all the methods are consistent with this.

b) Diced Data

Presented in Figure 5.5(a) is the bias error, averaged over all 106 “diced” data sets. The results are very similar to those obtained using the “sliced” data sets. The standard deviation of the bias error is presented in Figure 5.5(b) and when compared with the “sliced” data it apparent that the standard deviation for a number of the methods has substantially reduced. It can be seen that all single parameter fits do not perform as well as two parameter fits. The “diced” results reflect the true error in the model whereas the “sliced” results contain not only model errors but also errors due to trends in the data sets.

### 5.2.3 Predicted Historical Turbine Yield (Bias Error)

Whilst predicting the mean wind speed is important it is also crucial that its distribution is modelled correctly as this will have an impact upon energy yield calculations. All single parameter regression equations force the shape of the predicted distribution to be identical to the historical reference site. To a lesser extent two parameter regression equations also do the same and it is only the Matrix method which will preserve the distribution of the predictor site. To undertake this analysis the predicted wind speeds were used to generate an energy yield by using a typical power curve (Figure 5.6), and the normalised bias error is then derived as:

$$\mathrm{E}_E = \frac{E_{h,pred} - E_{h,act}}{E_{h,act}} \quad [16]$$

where $E_{h,pred}$ and $E_{h,act}$ are the predicted and actual energy yields for the historical period.

a) Sliced

Presented in Figures 5.7(a) and 5.7(b) are the average and standard deviation of the bias errors for each method respectively. With the exception of LSQ1 all the methods have a bias error of less than 3% and again the Matrix method has the smallest. By comparing Figures 5.4 and 5.7 it can be observed that a positive bias error in the wind speed is not always mirrored by similar bias error in the turbine yield, e.g. LSQ2 and OR2. This change in the sign of the bias errors is probably a manifestation of the shape of the wind speed distribution.

The standard deviations of the bias errors for all methods are in the range 8 to 11% and are consistent with the errors in the wind speed.

b) Diced

The average bias error, for “diced” data is presented in Figure 5.8(a) and is very similar to the results from using the “sliced” data (see Figure 5.7(a)). The standard deviation is presented in Figure 5.8(b) and this shows a reduction compared with the “sliced” data. These results clearly show that Method A and the Matrix method out perform all the other methods.

### 5.2.4 Distribution of Bias Errors

Presented in Figures 5.9 to 5.16 are the distributions of the bias errors for LSQ2, Methods A and B and the Matrix method for both “sliced” and “diced” data. From these figures it is clear that the bias errors are not symmetrically distributed for LSQ2 especially for those associated with the energy yield. The difference in distributions of the bias errors between wind speed and energy yield is another clear pointer that being able to predict the distribution along with its mean is crucial. Methods A and B perform better than LSQ2, but not as well as the Matrix method.

## *5.3 Variation with Concurrent Data Length*

Shown in Figures 5.17 to 5.24 and summarized in Tables 5.1 to 5.4 is the variation of the bias error and its standard deviation with the length of the concurrent data for the LSQ2, Methods A and B and the Matrix method. It is apparent from these figures that as the amount of data increases the standard deviation of the bias error reduces substantially but that the average bias error does not follow any clear trend.

| | Method (Sliced Data) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | LSQ2 | | Method B | | Method A | | Matrix | |
| Months | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) |
| 6 | 0.34 | 4.28 | -1.32 | 4.42 | 1.26 | 4.16 | -0.52 | 4.23 |
| 12 | 0.77 | 3.45 | -1.37 | 3.36 | 1.50 | 3.46 | 0.06 | 3.39 |
| 18 | 0.66 | 2.87 | -1.37 | 3.02 | 0.88 | 2.53 | 0.41 | 2.81 |
| 24 | 0.50 | 2.19 | -0.47 | 2.56 | 0.70 | 2.17 | 0.26 | 2.21 |

Table 5.1 – Variation of Wind Speed Bias Error and Standard Deviation with Concurrent Data Length (sliced)

| | Method (Sliced Data) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | LSQ2 | | Method B | | Method A | | Matrix | |
| Months | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) |
| 6 | -3.48 | 9.46 | -2.14 | 8.44 | 1.65 | 8.74 | -0.85 | 7.75 |
| 12 | -2.98 | 8.19 | -2.58 | 5.92 | 1.49 | 6.44 | -0.01 | 6.44 |
| 18 | -5.78 | 7.81 | -1.66 | 5.34 | 1.03 | 6.03 | 0.80 | 5.87 |
| 24 | -5.13 | 5.45 | -0.68 | 5.06 | -0.21 | 4.93 | 0.67 | 4.97 |

Table 5.2 – Variation of Energy Bias Error and Standard Deviation with Concurrent Data Length (sliced)

| | **Method (Diced Data)** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **LSQ2** | | **Method B** | | **Method A** | | **Matrix** | |
| Months | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) |
| 6 | 0.77 | 1.74 | -1.65 | 3.20 | 1.16 | 1.98 | -0.06 | 1.56 |
| 12 | 0.93 | 1.46 | -1.51 | 2.72 | 1.31 | 1.79 | 0.20 | 1.33 |
| 18 | 0.64 | 1.31 | -1.06 | 2.95 | 0.91 | 1.62 | 0.43 | 1.16 |
| 24 | 0.38 | 1.00 | -0.60 | 1.52 | 0.48 | 1.02 | 0.20 | 0.92 |

Table 5.3 – Variation of Wind Speed Bias Error and Standard Deviation with Concurrent Data Length (diced)

| | **Method (Diced Data)** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **LSQ2** | | **Method B** | | **Method A** | | **Matrix** | |
| Months | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) | Bias Error (%) | Bias SD (%) |
| 6 | -2.75 | 5.78 | -2.56 | 3.20 | 0.97 | 3.57 | -0.22 | 3.06 |
| 12 | -2.54 | 5.27 | -2.62 | 2.72 | 1.24 | 3.26 | 0.38 | 2.53 |
| 18 | -5.56 | 5.57 | -1.04 | 2.95 | 0.59 | 2.84 | 0.78 | 2.40 |
| 24 | -5.09 | 3.37 | -0.75 | 1.52 | -0.22 | 2.93 | 0.58 | 2.18 |

Table 5.4 – Variation of Energy Bias Error and Standard Deviation with Concurrent Data Length (diced)

### *5.4 Variation with Correlation Coefficient*

Shown in Figures 5.25 to 5.28 is the variation of the bias standard with the product-moment correlation coefficient, $r$ , for the LSQ2, Methods A and B and the Matrix method for both the sliced and diced data sets. The coefficient has been calculated using all the concurrent data prior to sectoring. All methods show a similar result, in that the standard deviation reduces for both quantities as $r$ increases. The reduction in the wind speed standard deviation is far more pronounced than the energy yield.

## 6.0 CONCLUSIONS AND RECOMMENDATIONS

A thorough review of a number of MCP techniques has been undertaken from which the following conclusions can be drawn:

a) All two parameter fits, the ratio of means and the Matrix methods are unbiased estimators in that they preserve the means of the data sets. Single parameters fits are biased estimators although the extent to which this impacts upon the errors in the MCP process is not clear.

b) The standard deviation of the bias error, which can be taken as an indicator of the accuracy of the MCP process, is 5% on wind speed and 9% on energy yield. As discussed, these results are contaminated by trends in the data, and by using a “diced” data set, it is possible to show that the underlying error in fitting techniques is typically 2% for wind speed and 5% for energy yield, with Method A and the Matrix method performing substantially better than the other methods.

c) For concurrent data lengths greater than 12 months the error drops to less than 3.5% on wind speed and 7.0% on energy.

d) The distribution of the bias errors for all methods, other than the Matrix method, is asymmetric and non Gaussian; this is likely to be an important consideration in the selection of an MCP technique.

e) Based on the statistical techniques used in this report it can be concluded that the Matrix method should be used in preference to the other methods reviewed. Method B performs well but it is questionable whether a single parameter fit coupled with an arbitrary censoring of the data is appropriate. Furthermore, the technique fails to back predict and has a large bias standard deviation when analysed using the “diced” data set. Method A performs to a level comparable or slightly better than Method B. However, the method, as in common with all two parameter fits is susceptible to the frequency of low wind speeds which can significantly distort the regression fit. Also the large offset, which two parameter fits can potentially introduce, is physically unrealistic.

f) Given the level of uncertainty of the MCP procedure it is questionable whether or not it will give any genuine benefit where more than 24 months of continuous data has been collected.

The purpose of this report is to evaluate a number of MCP techniques. However, the results of this report should not be taken as a recipe for predicting the long term wind speed distribution at a potential wind farm. Prior to using any technique the data needs to be carefully scrutinised taking into consideration other factors such as; quality, length, stability of reference site, correlation coefficient etc. and only once this has been undertaken can the most appropriate technique be used. In some cases an MCP technique may not be the best solution and the investigator will have to rely solely on the measured data.

## 7.0 REFERENCES

[1] Bass, J.H., Rebbeck, M., Landberg, L., Cabre, M. and Hunter, A., 2000, "An Improved Measure Correlate Predict Algorithm for the Prediction of the Long Term Wind Climate in Regions of Complex Environment".

[2] Bass JH 1999, 'A Description of the RES MCP Method for Long Term Wind Speed Prediction', RES Report 504/RES/2001 Issue 2, 1999.

[3] Garrad Hassan, 2003, "Personal communication from Marc LeBlanc".

[4] Fuller, W.A., 1987, "Measurement Error Models", Wiley, New York.

[5] Carroll, R. J., and D. Ruppert, 1996, "The Use and Misuse of Orthogonal Regression in Linear Errors-in-Variables Models", The American Statistician, 50, 1-6.

[6] Numerical Recipes in Fortran, 2001, Cambridge University Press.

[7] Rebbeck, M., 1996, "A comparison of various measure-correlate-predict techniques for wind resource assessment", MSc Thesis, Dept. of Electronic and Electrical Engineering, Loughborough University.

[8] Mortimer, A. A., 1994, "A new correlation/prediction method for potential wind farm sites", Proc. of the Sixteenth BWEA Wind Energy Conference, Ed. G.Elliot.

# ANNEX A - ORTHOGONAL REGRESSION

## *A.1 No Errors in Variables*

Assume that two variables, *Y* and X, are theoretically linearly related:

$$Y = \alpha + \beta X \quad [A.1]$$

where $\alpha$ is the intercept, $\beta$ is the slope. One most commonly used way to estimate $\beta$ is to use ordinary least squares which minimizes the vertical distance between the observations and the fitted line. However, the ordinary least squares regression methodology will not be valid when the dependent and independent variables cannot be pre-determined, or when there are measurement errors in variables. In such cases the orthogonal regression method is thought to be more applicable. The orthogonal regression estimators are obtained by minimizing directly the distance between the observations and the fitted line, and are:

$$\hat{\alpha} = \overline{Y} - \hat{\beta}\overline{X} \quad [A.2]$$

$$\hat{\beta} = \frac{m_{YY} - m_{XX} + \sqrt{(m_{YY} - m_{XX})^2 + 4m_{XY}^2}}{2m_{XY}} \quad [A.3]$$

where $m_{XY}$ is the covariance between *X* and *Y*, and $m_{XX}$ and $m_{YY}$ the variances of *X* and *Y* respectively.

The general formula for calculating a sample covariance matrix is

$$m_{ij} = \frac{\sum (z_i - \overline{z_i})(z_j - \overline{z_j})}{N-1} \quad [A.4]$$

Alternatively if we assume that the two variables, *Y* and X, are theoretically linearly related by:

$$Y = \beta X \quad [A.5]$$

where $\beta$ is the slope. Minimisation of the perpendicular distance to the line results in an estimate for the gradient of the line of

$$\hat{\beta} = \frac{n_{yy} - n_{xx} + \sqrt{(n_{yy} - n_{xx})^2 + 4n_{xy}^2}}{2s_{xy}} \quad [A.6]$$

It should be noted that the covariance's are with respect to the origin, not the mean as is the general practice, e.g.

$$n_{ij} = \frac{\sum z_i z_j}{N-1} \quad [A.7]$$

A consequence of this formulation is that fitted line will not pass through the mean, $\overline{X}, \overline{Y}$ of the data as it is forced to pass through the origin and as such is a biased estimator. Furthermore, it is clear from equation [A.6] that, whenever $n_{xx} = n_{yy}$, the orthogonal slope estimator will always equal unity. Therefore, whenever the variances of the two variables are close to each other, no matter whether the two variables are related, the orthogonal regression will always render a significant and close to unity relationship between the two variables. Also the method suffers from the problem that the estimated relationship will be based on the measurement units used for the variables. For example, one can always find a significant equal proportionate relationship between two variables by simply changing the measurement unit of one of the variables (i.e. rescaling).

### *A.2 Errors in Variables*

Since the estimators of orthogonal regression are obtained by minimizing the perpendicular distance between the observations and the fitted line, the orthogonal regression is especially applicable to the case when the independent and dependent variables cannot be pre-determined. Orthogonal regression has also been advocated when there are errors in the variables. Assume that *x* and *y* are variables measured with errors, that is,

$$x = X + \varepsilon_x \quad \text{[A.8]}$$

$$y = Y + \varepsilon_y \quad \text{[A.9]}$$

where $\varepsilon_x$ and $\varepsilon_y$ are measurement errors for *x* and *y* respectively.

$$p_{xx} = m_{XX}^2 + \sigma_{\varepsilon x}^2 \quad \text{[A.10]}$$

$$p_{yy} = m_{YY}^2 + \sigma_{\varepsilon y}^2 \quad \text{[A.11]}$$

The orthogonal regression slope estimator when there are errors in the variables is [Fuller 1987]:

$$\widehat{\beta} = \frac{p_{yy} - \lambda p_{xx} + \sqrt{(p_{yy} - \lambda p_{xx})^2 + 4\lambda p_{xy}^2}}{2p_{xy}} \; ; \; \hat{\alpha} = \bar{y} - \widehat{\beta}\bar{x} \quad \text{[A.12]}$$

$$\lambda = \frac{\sigma_{\varepsilon y}^2}{\sigma_{\varepsilon x}^2} \quad \text{[A.13]}$$

where $p_{xy}$ is the sample covariance between x and y, and $p_{xx}$, $p_{yy}$, $\sigma_{\varepsilon x}^2$ and $\sigma_{\varepsilon y}^2$ the variances of *x*, *y*, $\varepsilon_x$ and $\varepsilon_y$ respectively. In some text books [Numerical Recipes 2001] the above method is sometimes referred to as the York Method. It should be noted that when $\lambda >> 1$ then the formula for the gradient reduces to that of simple linear regression.

A further extension to orthogonal regression is to consider not only errors in the variables but also in the equation itself [Carroll 1996]. If the variance of the error in equation [A.1] can be represented by $\sigma_{uu}$ then equation [A.13] can written as

$$\lambda = \frac{\sigma_{uu}^2 + \sigma_{\varepsilon y}^2}{\sigma_{\varepsilon x}^2} \quad \text{[A.14]}$$

As it is likely that error in the equation is significantly greater than either of the errors in measurement of the dependent or independent variables then $\lambda >> 1$. This suggests that in practice the orthogonal regression fitted line will tend towards that derived from ordinary least squares.

### *A.3 Method A*

A variant of the errors in variables orthogonal regression model is through the minimisation of the function

$$S = \frac{\sum(y_i - \beta x_i - \alpha)^2}{(\beta \sin\phi + \cos\phi)^2} \qquad [A.15]$$

where

$$\phi = \tan^{-1}\left(\beta_0 \frac{\sigma_{\varepsilon x}^2}{\sigma_{\varepsilon y}^2}\right) \qquad [A.16]$$

$$\beta_0 = \frac{\sigma_{xy}^2}{\sigma_{xx}^2} \qquad [A.17]$$

Through partial differentiation of equation [A15] it is possible to solve for the gradient of the line:

$$\hat{\beta} = \frac{\sin\phi \sum(y_i - \bar{y})^2 + \cos\phi \sum y_i(x_i - \bar{x})}{\sin\phi \sum y_i(x_i - \bar{x}) + \cos\phi \sum(x_i - \bar{x})^2} \; ; \quad \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} \qquad [A.18]$$

The difference between this method and standard orthogonal regression analysis is that the angle the residual makes to the fitted line is fixed. Replacing $\beta_0$ in equation [A16] with by $\beta$ would result in the two methods becoming identical.

The parameters $\sigma_{\varepsilon x}$ and $\sigma_{\varepsilon y}$ are estimated from the application of

$$\sigma_{\varepsilon x} = \frac{\sum(x_i - \overline{x_T})^2}{N-1} \qquad [A.19]$$

$$\sigma_{\varepsilon y} = \frac{\sum(y_i - \overline{y_T})^2}{N-1} \qquad [A.20]$$

where $\overline{x_T}$ and $\overline{y_T}$ are rolling averages over a period time, *T*. and is typically three hours. The resulting $\sigma_{\varepsilon x}$ and $\sigma_{\varepsilon y}$ are then binned by direction.

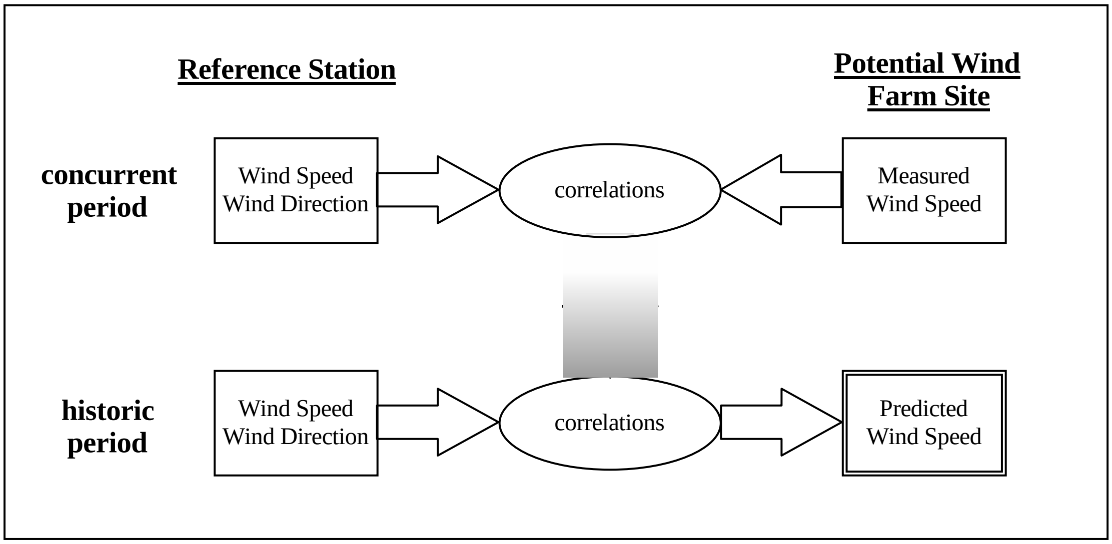


Figure 2.1 – Methodology of MCP process

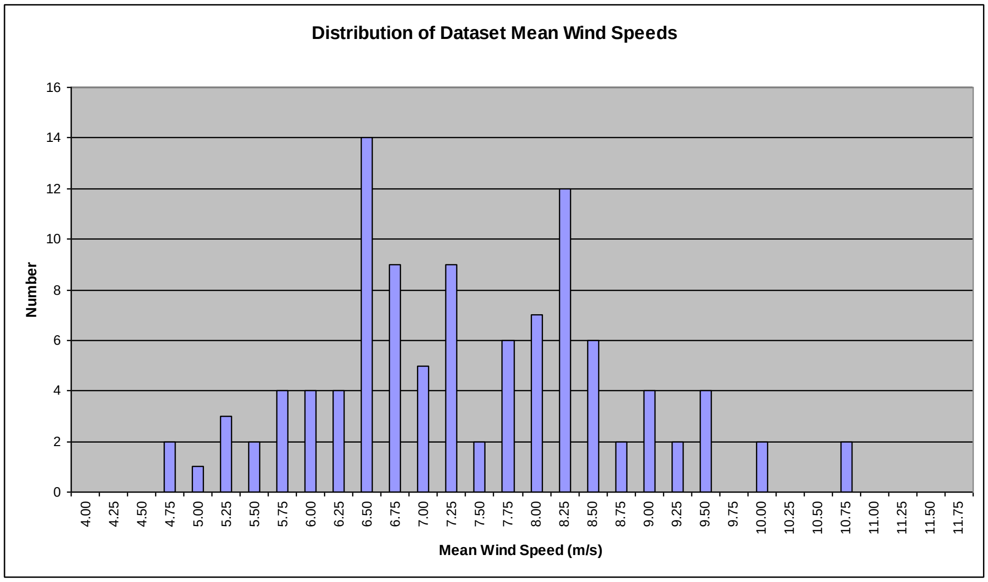


Figure 3.1 – Distribution of mean wind speeds within dataset

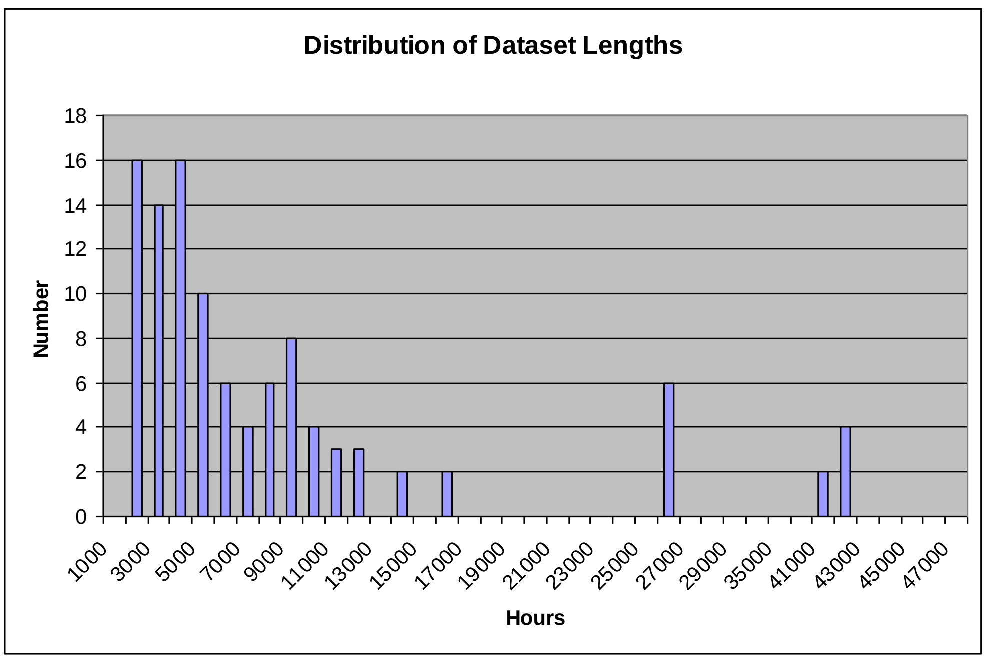


Figure 3.2 – Distribution of record lengths within dataset

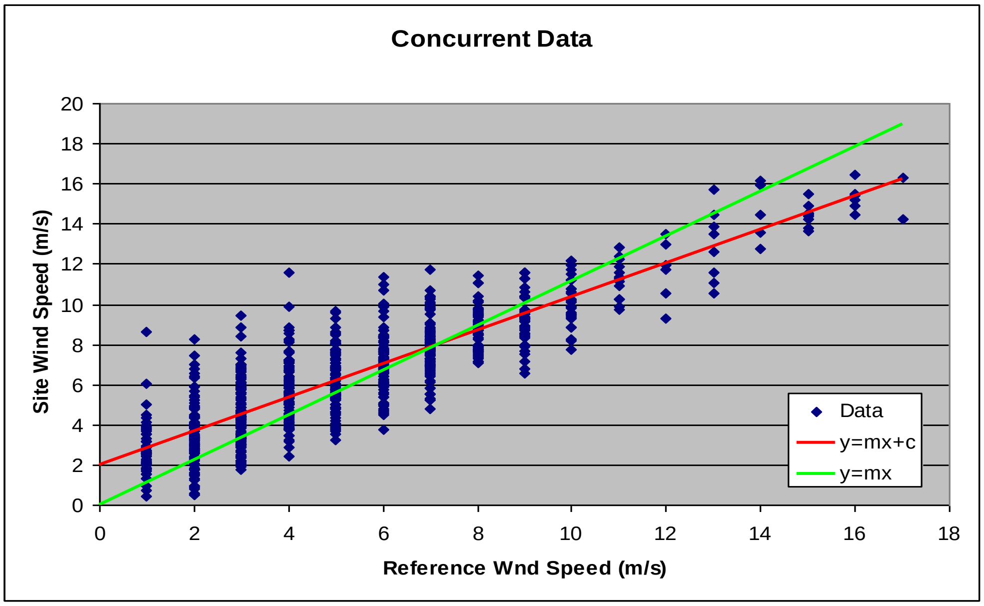


Figure 4.1 – Typical scatter plot of concurrent data for a wind direction sector

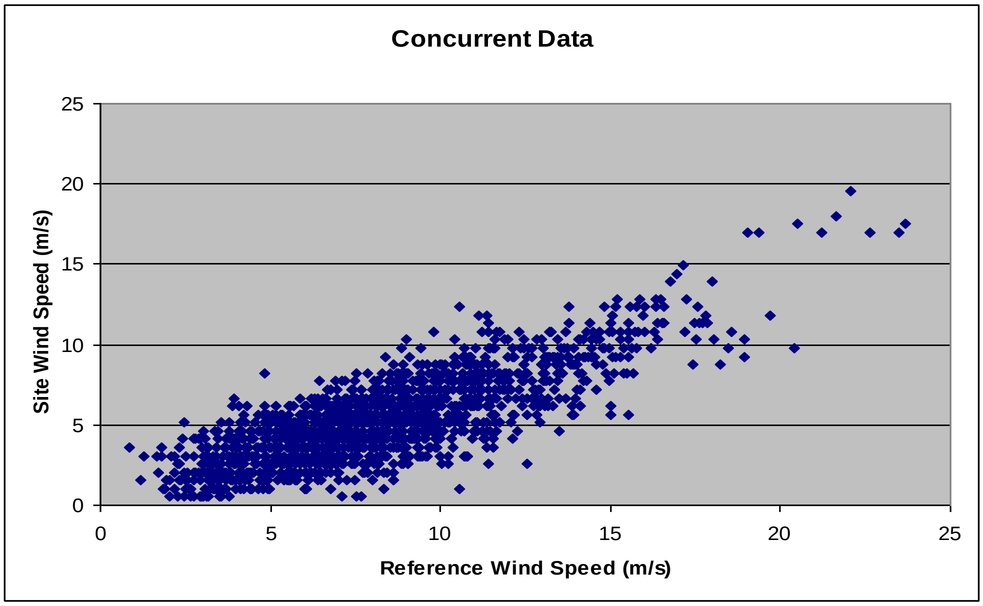


Figure 4.2 – Typical scatter plot of concurrent data for a wind direction sector

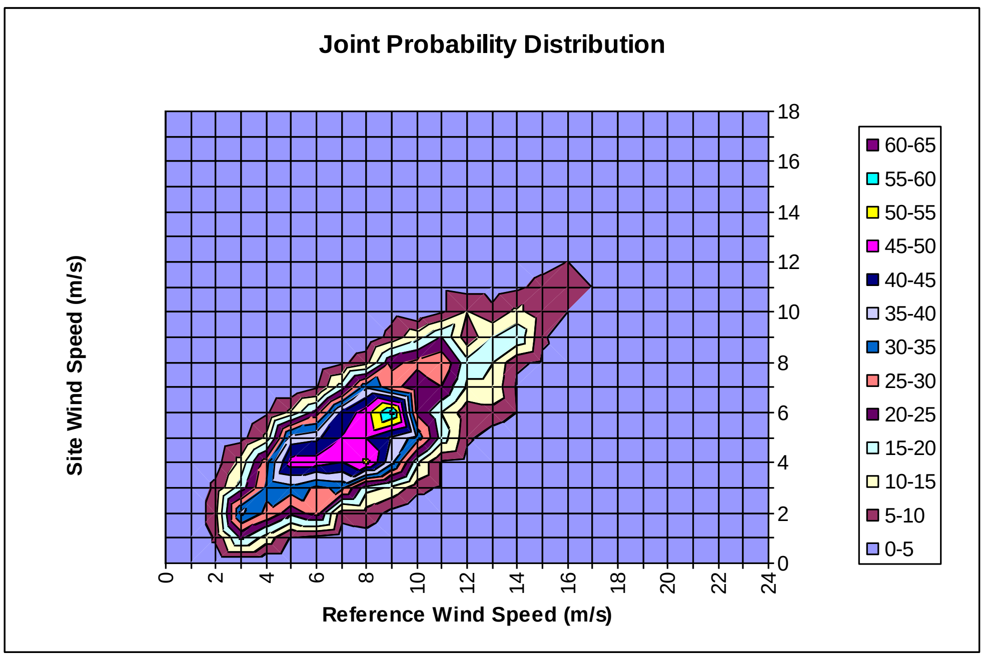


Figure 4.3 – Example of joint probability distribution of concurrent data for a wind direction sector (legend denotes the number of points within a bin)

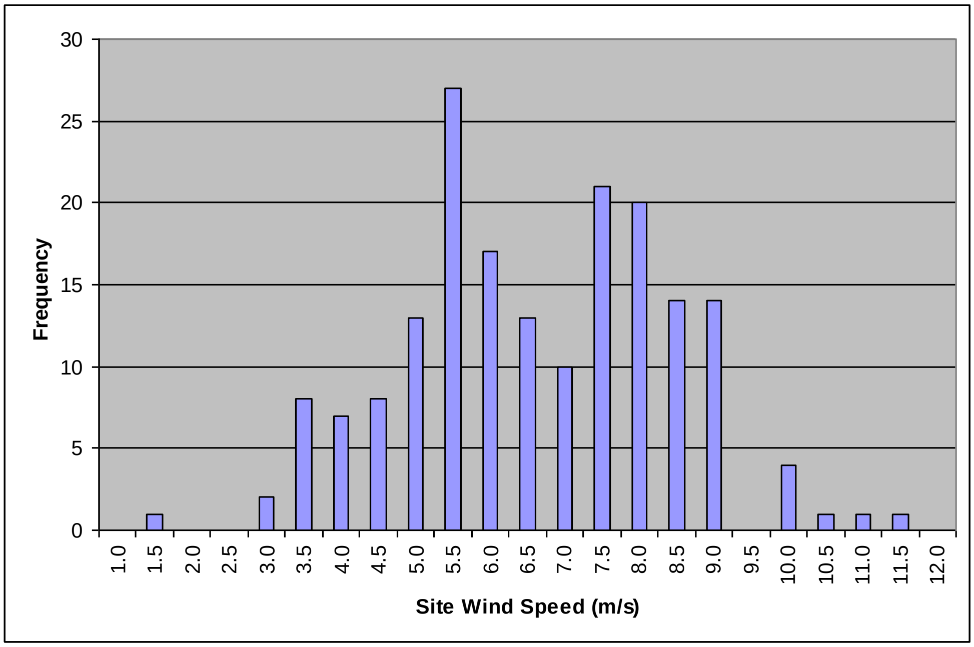


Figure 4.4 – Probability distribution of site wind speed for a given reference wind speed and direction sector

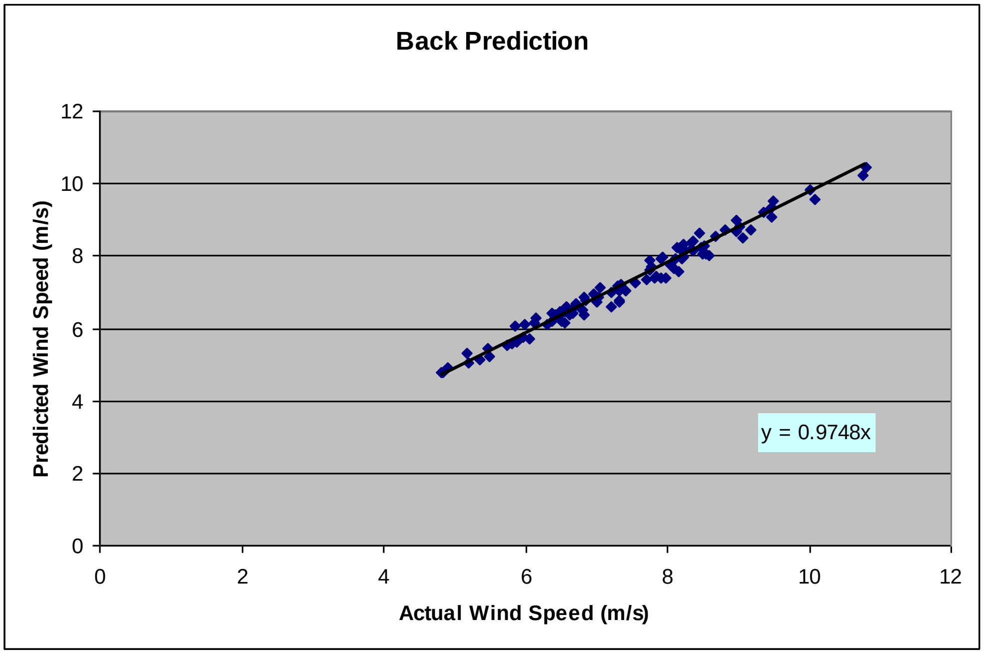


Figure 5.1 – Method B

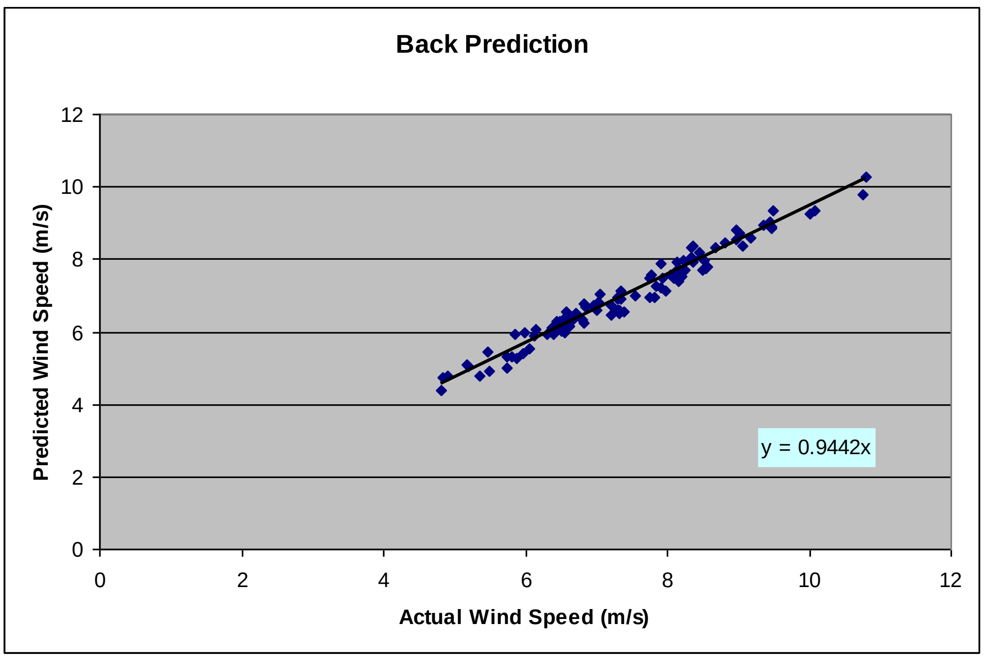


Figure 5.2 – LSQ ( $y = \beta x$ ) Method

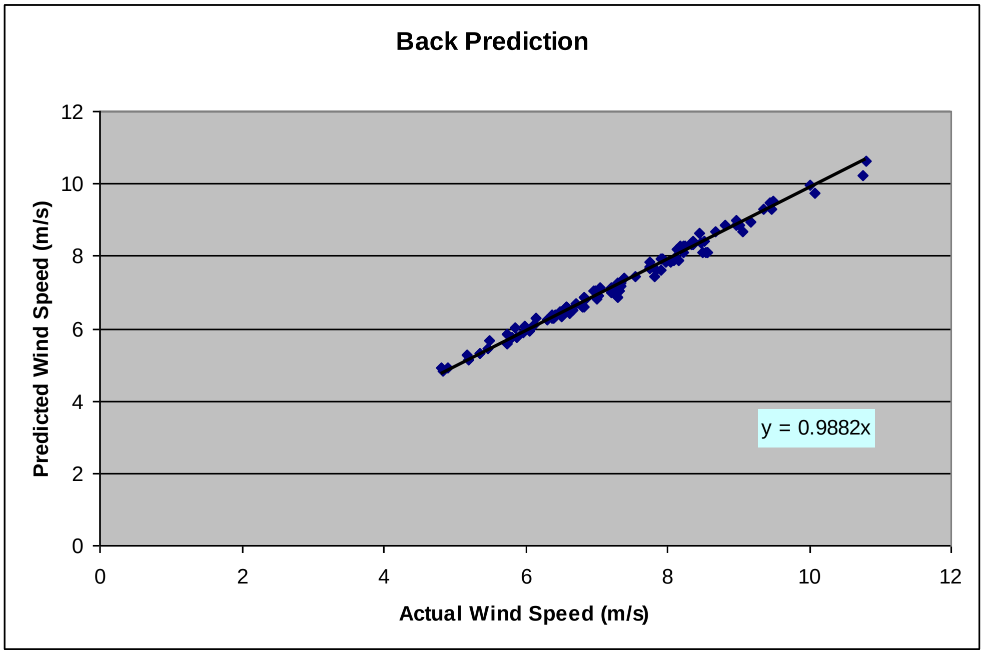


Figure 5.3 – Orthogonal ( $y = \beta x$ ) Method

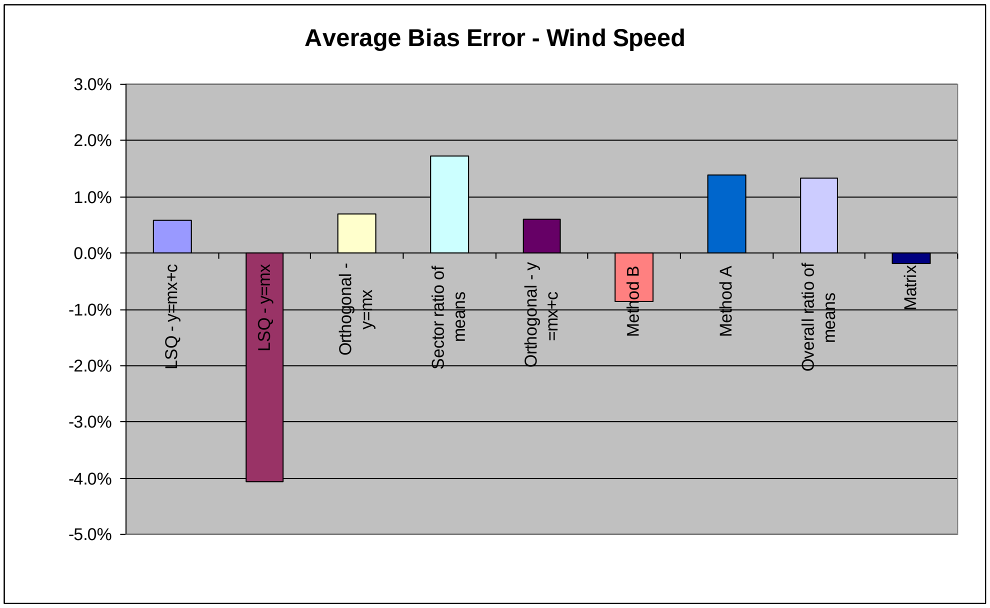


Figure 5.4(a) – Average bias error (sliced)

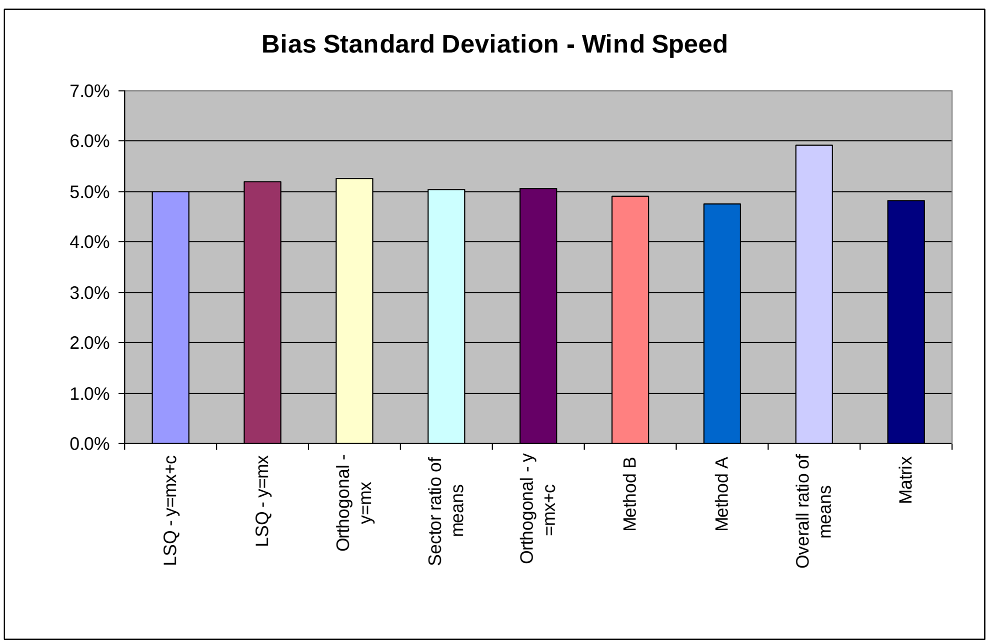


Figure 5.4(b) – Standard deviation of bias error (sliced)

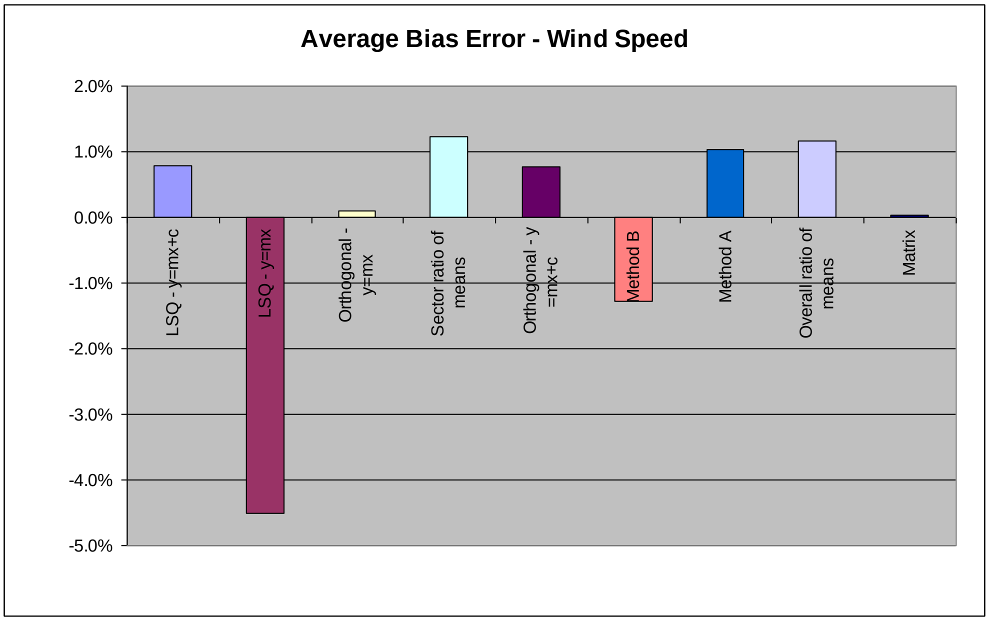


Figure 5.5(a) – Average bias error (diced)

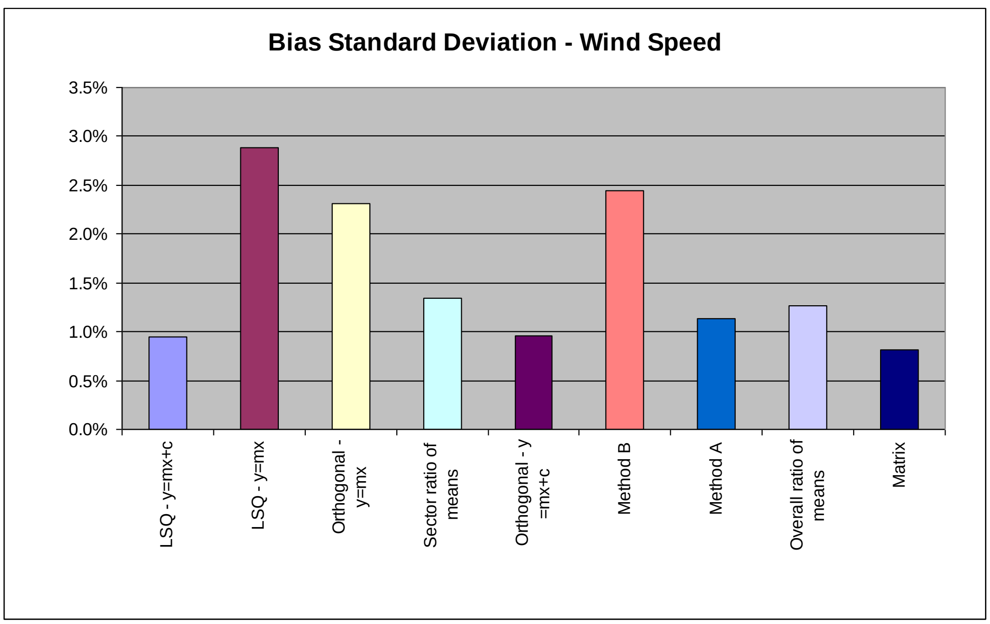


Figure 5.5(b) – Standard deviation of bias error (diced)

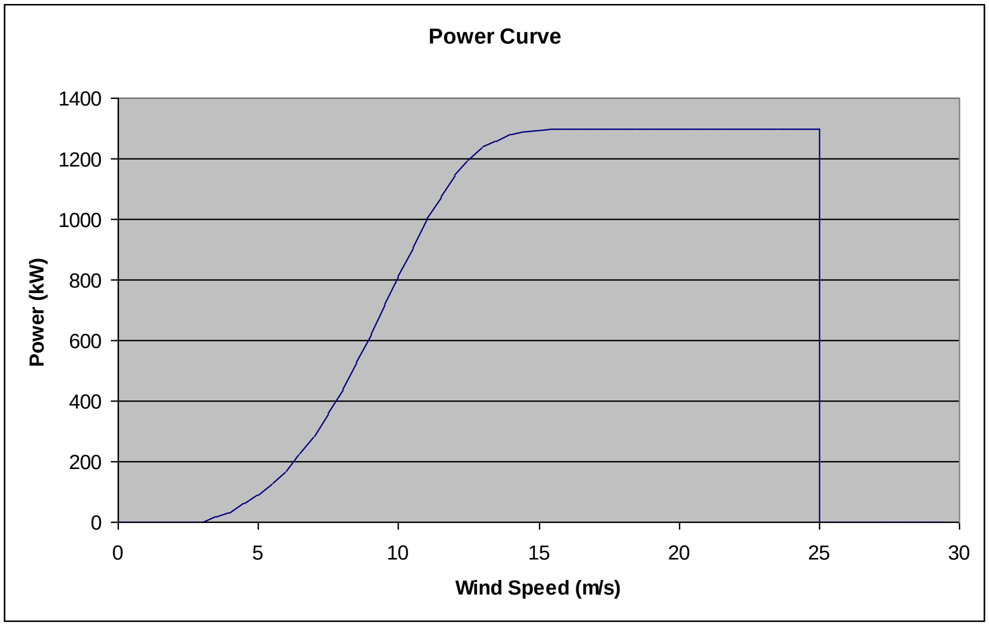


Figure 5.6 – Turbine power curve

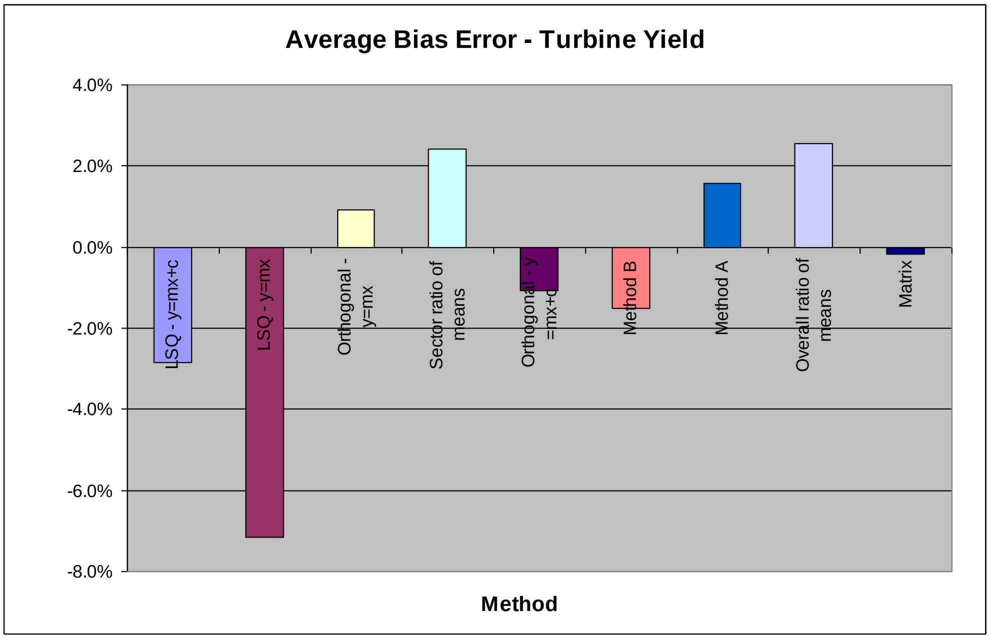


Figure 5.7(a) – Average bias error (sliced)

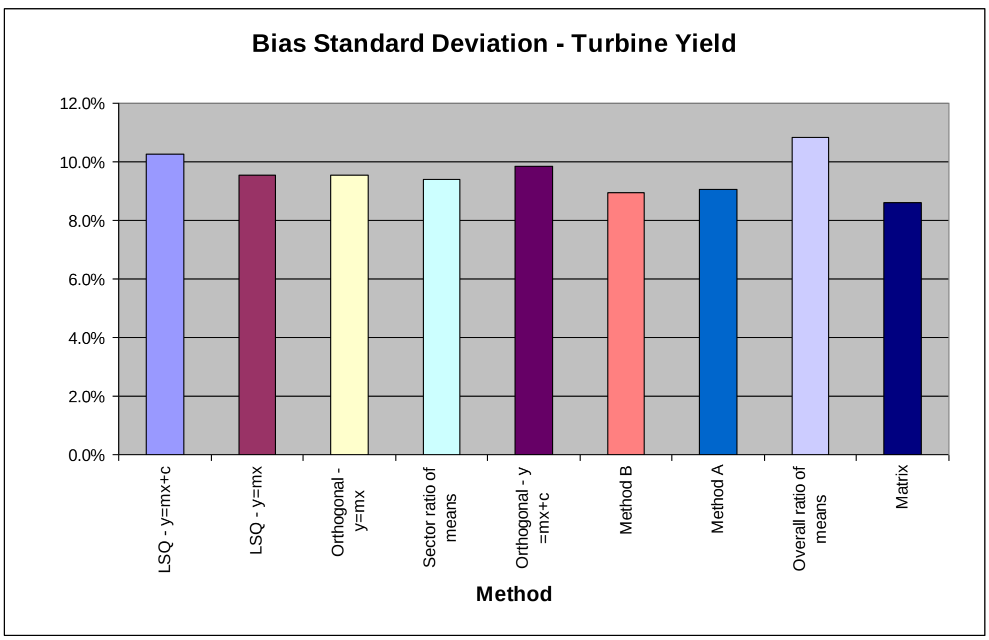


Figure 5.7(b) – Standard deviation of bias error (sliced)

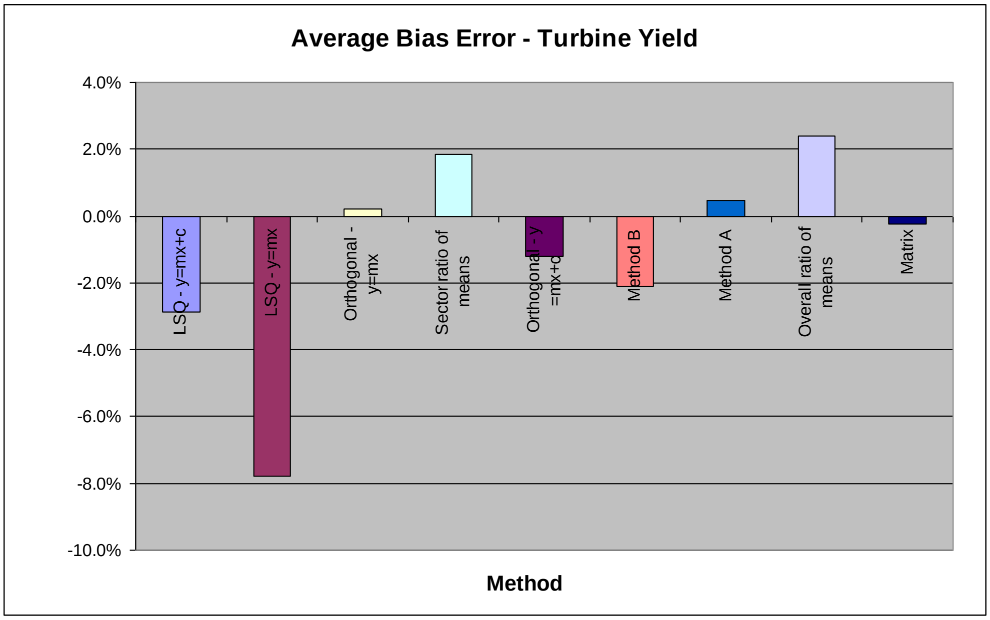


Figure 5.8(a) – Average bias error (diced)

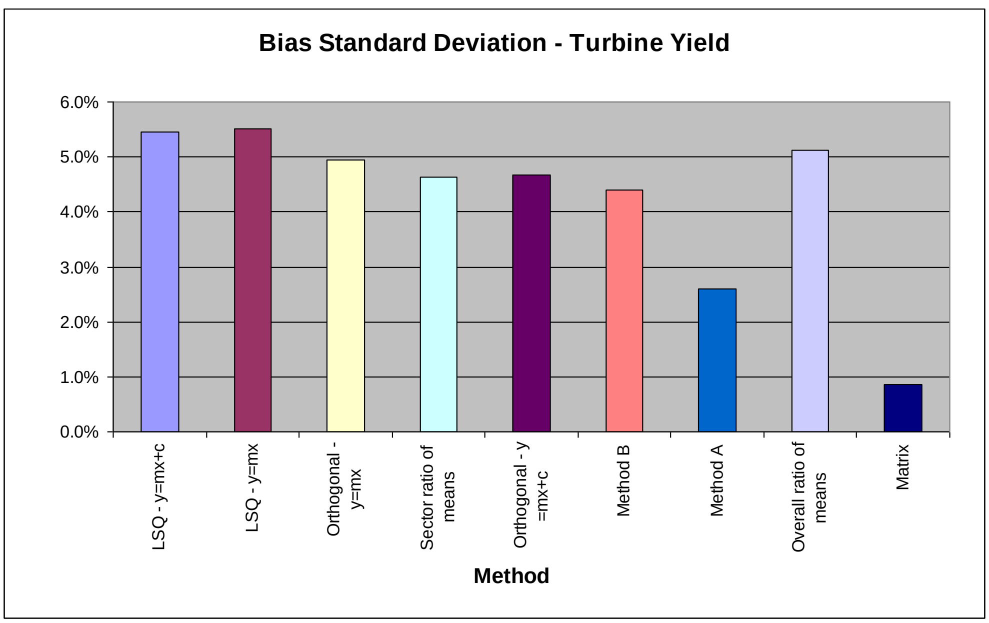


Figure 5.8(b) – Standard deviation of bias error (diced)

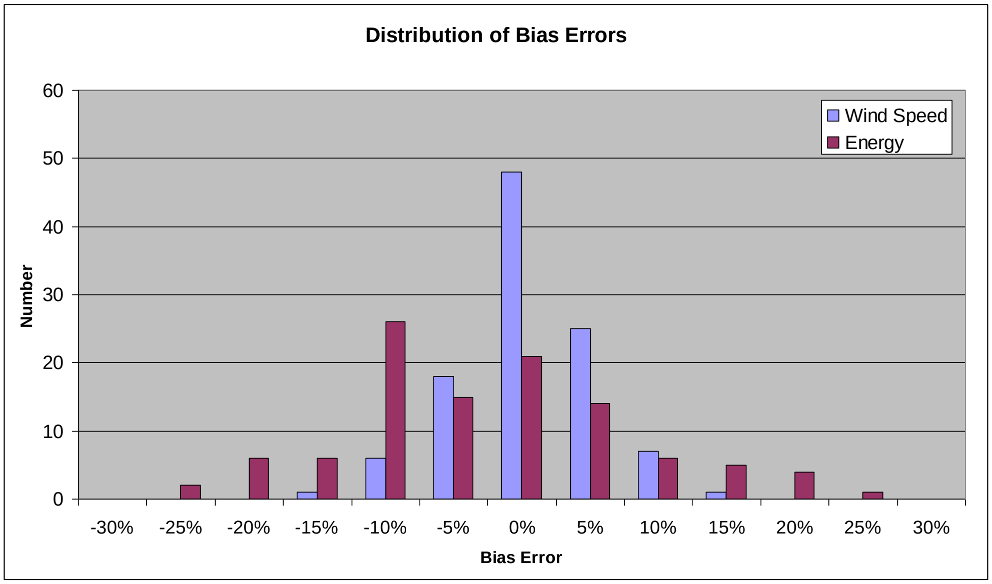


Figure 5.9(a) – Distribution of bias errors for LSQ2 (sliced)

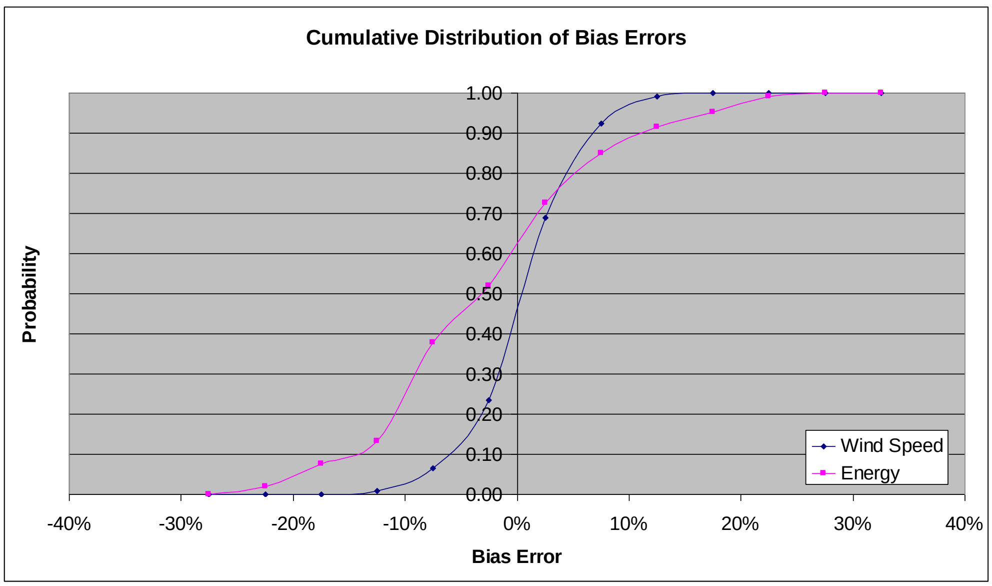


Figure 5.9(b) – Cumulative distribution of bias errors for LSQ2 (sliced)

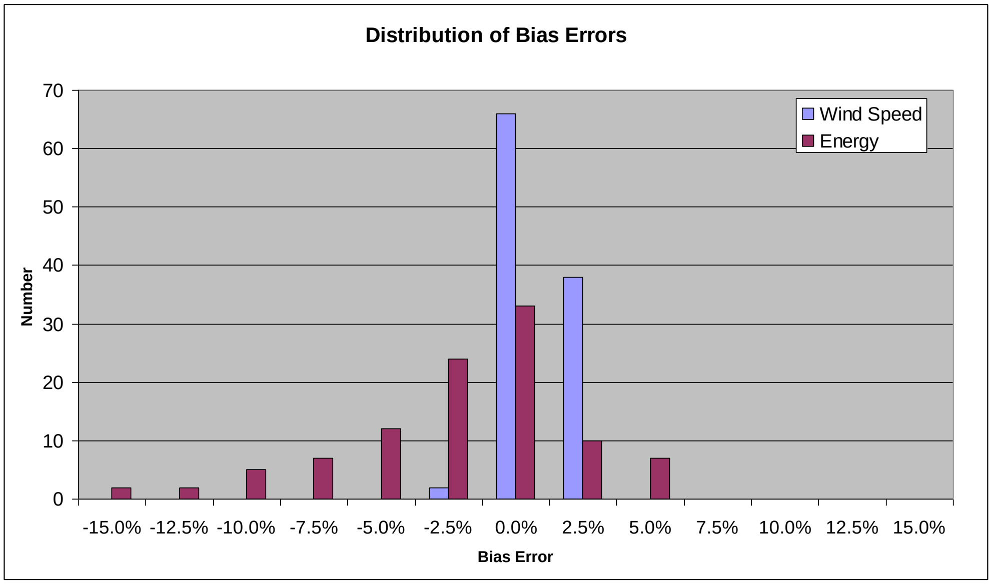


Figure 5.10(a) – Distribution of bias errors for LSQ2 (diced)

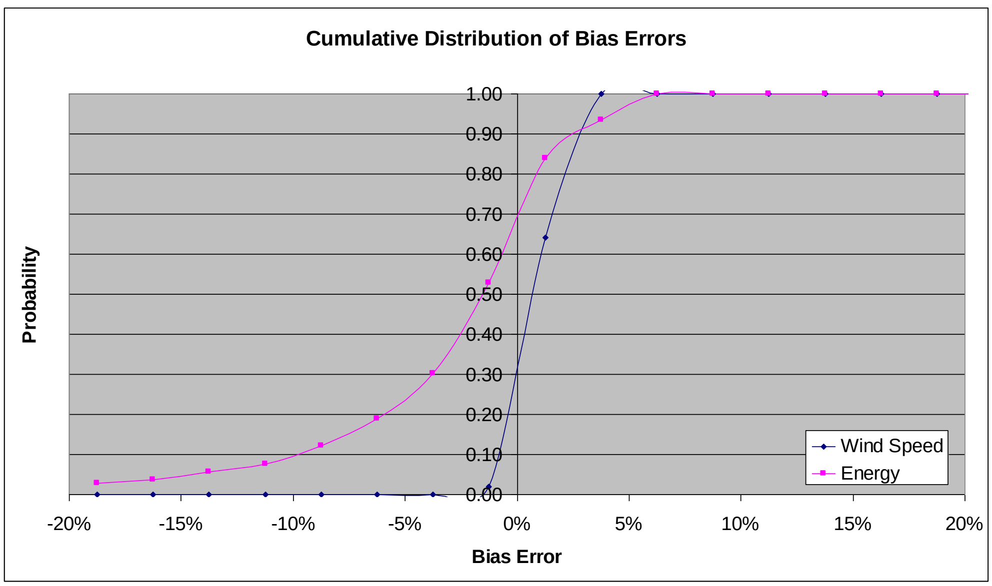


Figure 5.10(b) – Cumulative distribution of bias errors for LSQ2 (diced)

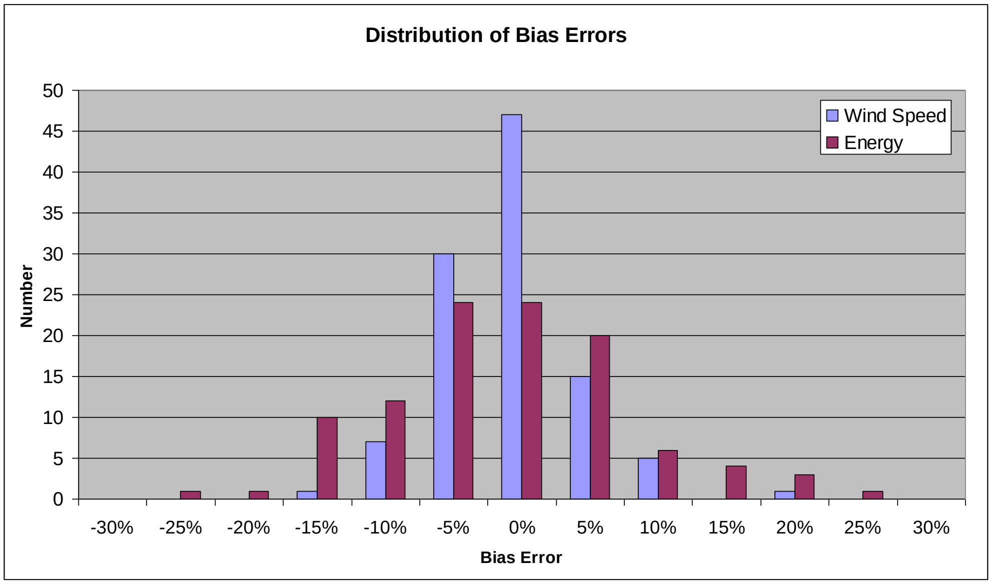


Figure 5.11(a) – Distribution of bias errors for Method B (sliced)

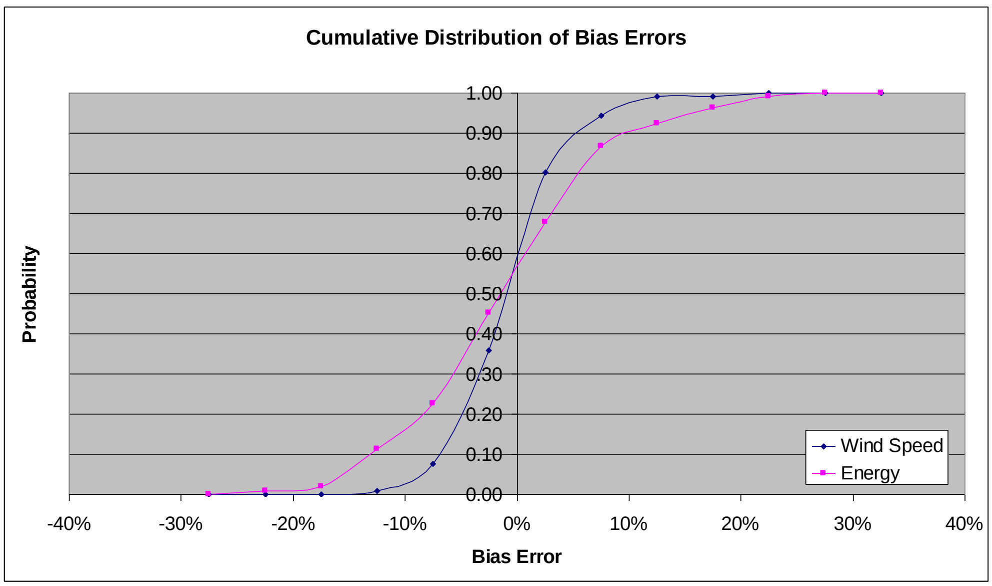


Figure 5.11(b) – Cumulative distribution of bias errors for Method B (sliced)

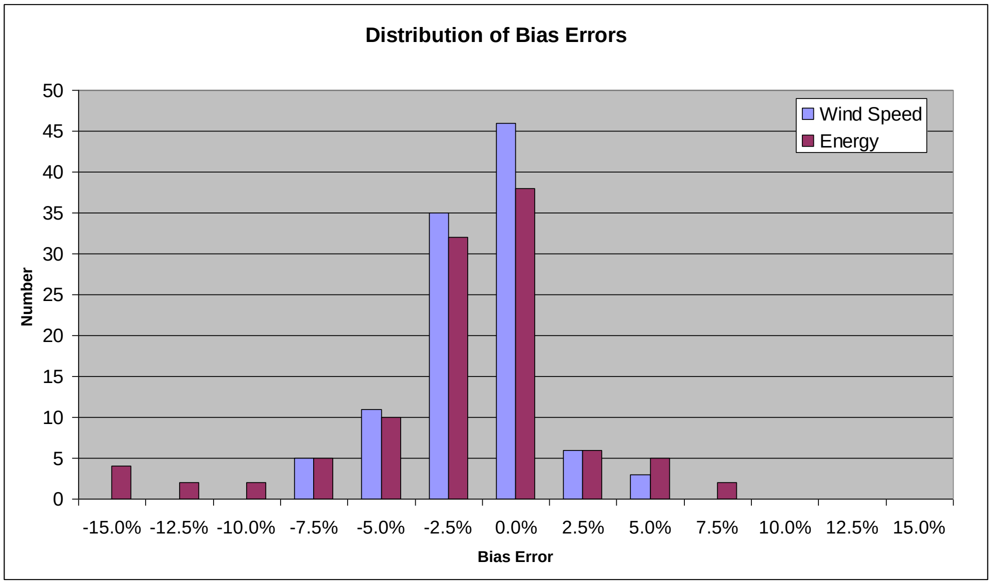


Figure 5.12(a) – Distribution of bias errors for Method B (diced)

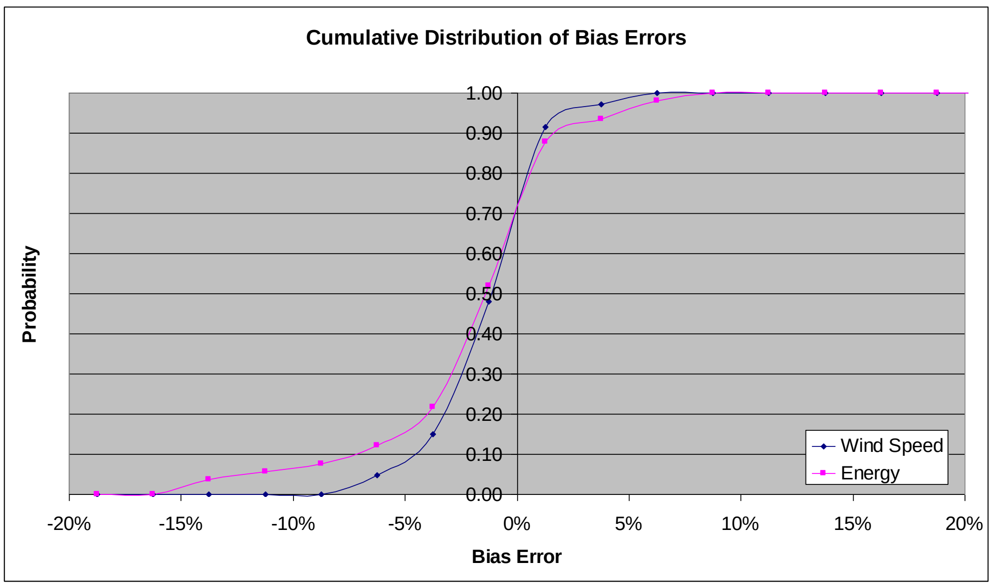


Figure 5.12(b) – Cumulative distribution of bias errors for Method B (diced)

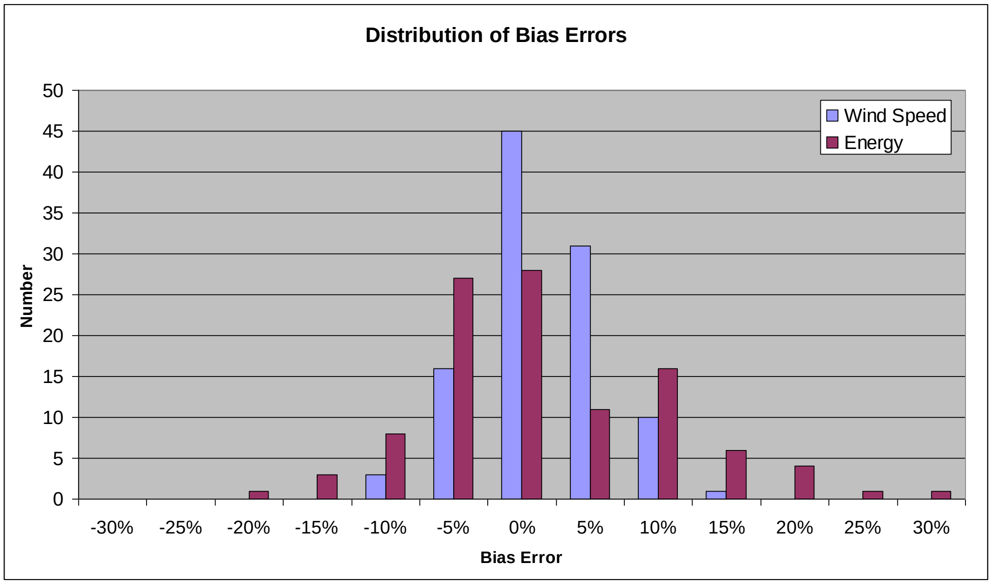


Figure 5.13(a) – Distribution of bias errors for Method A (sliced)

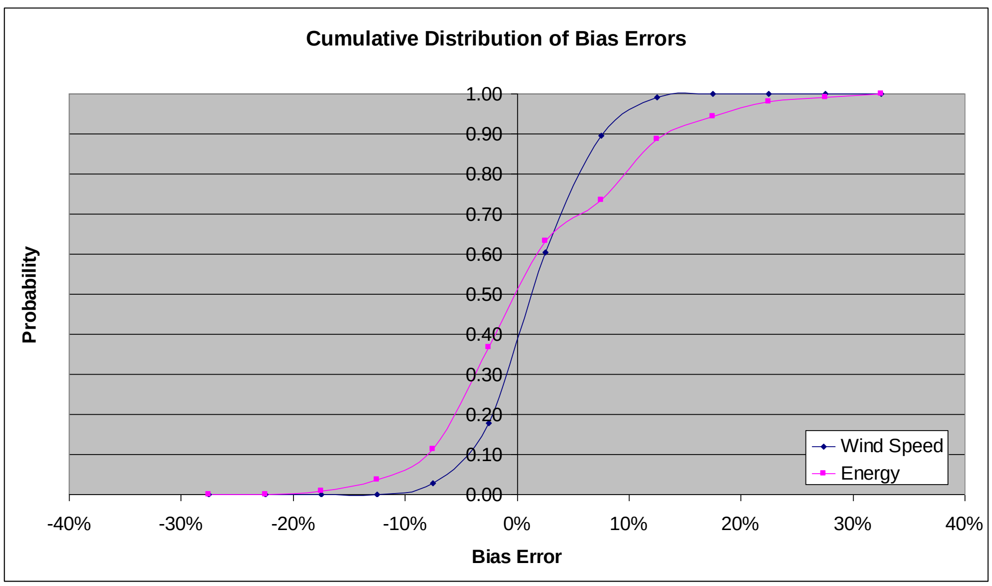


Figure 5.13(b) – Cumulative distribution of bias errors for Method A (sliced)

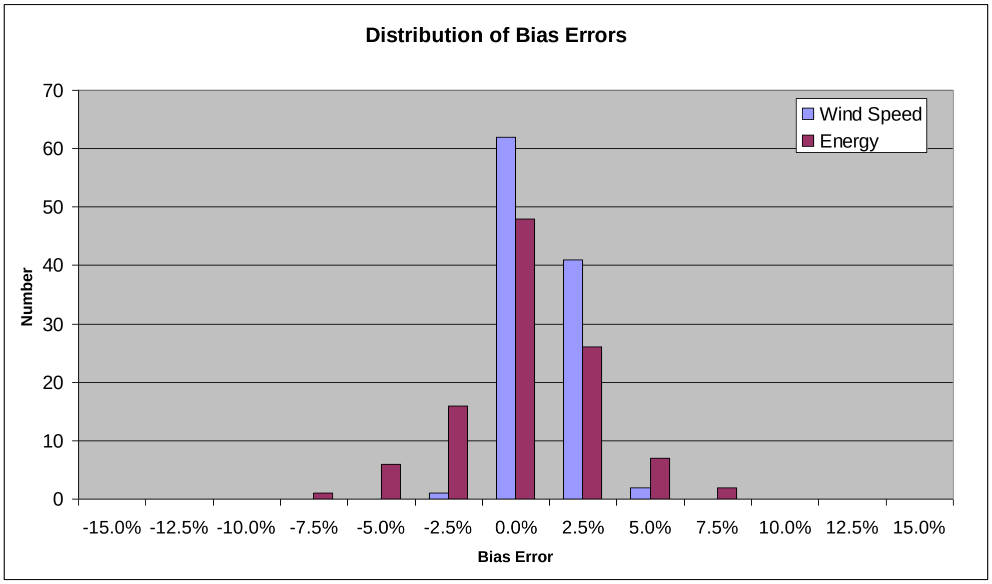


Figure 5.14(a) – Distribution of bias errors for Method A (diced)

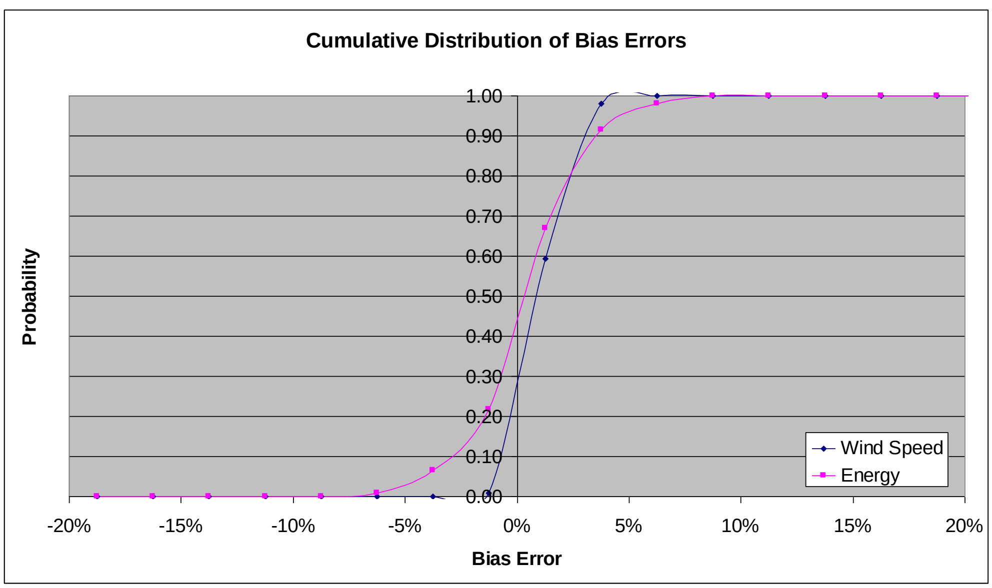


Figure 5.14(b) – Cumulative distribution of bias errors for Method A (diced)

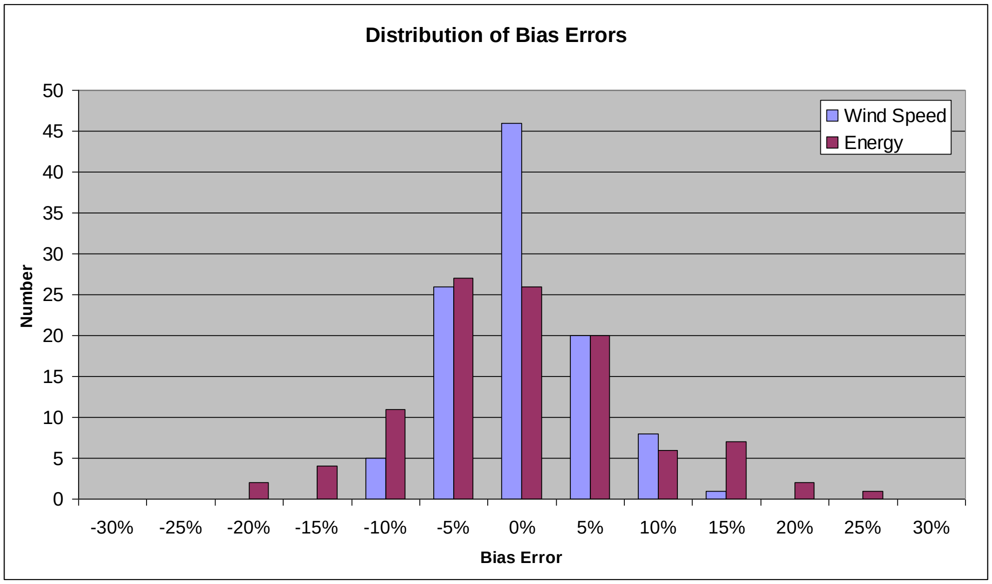


Figure 5.15(a) – Distribution of bias errors for Matrix (sliced)

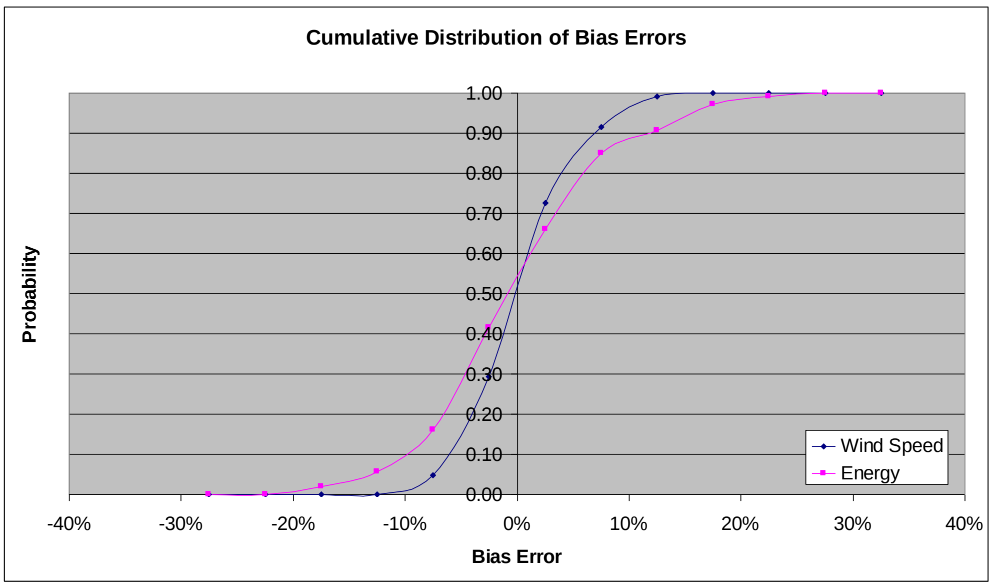


Figure 5.15(b) – Cumulative distribution of bias errors for Matrix (sliced)

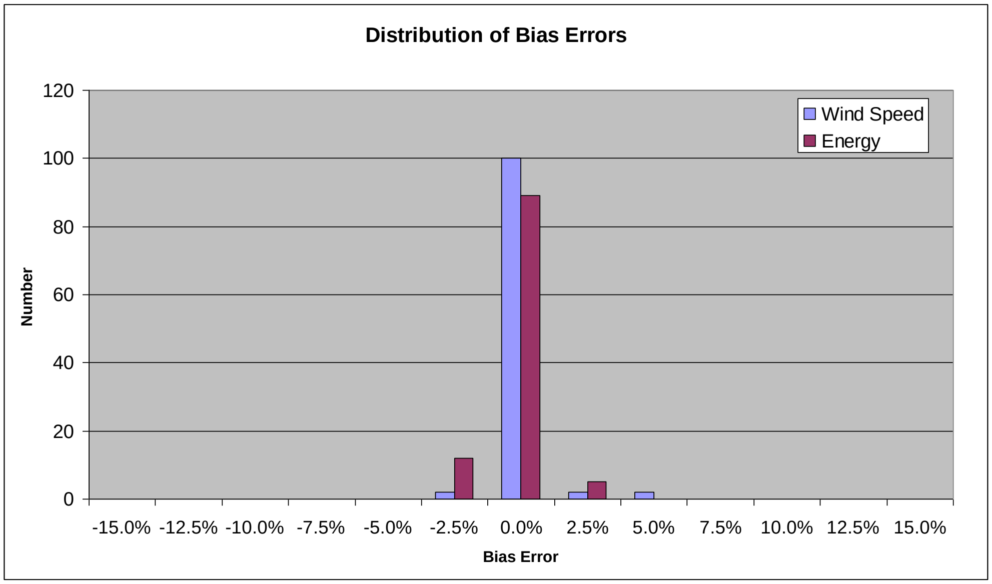


Figure 5.16(a) – Distribution of bias errors for Matrix (diced)

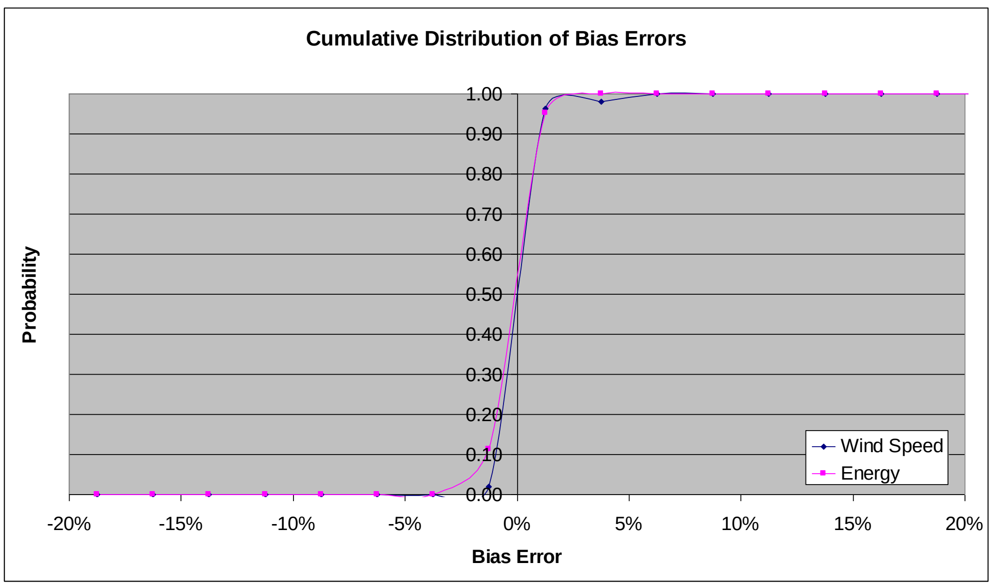


Figure 5.16(b) – Cumulative distribution of bias errors for Matrix (diced)

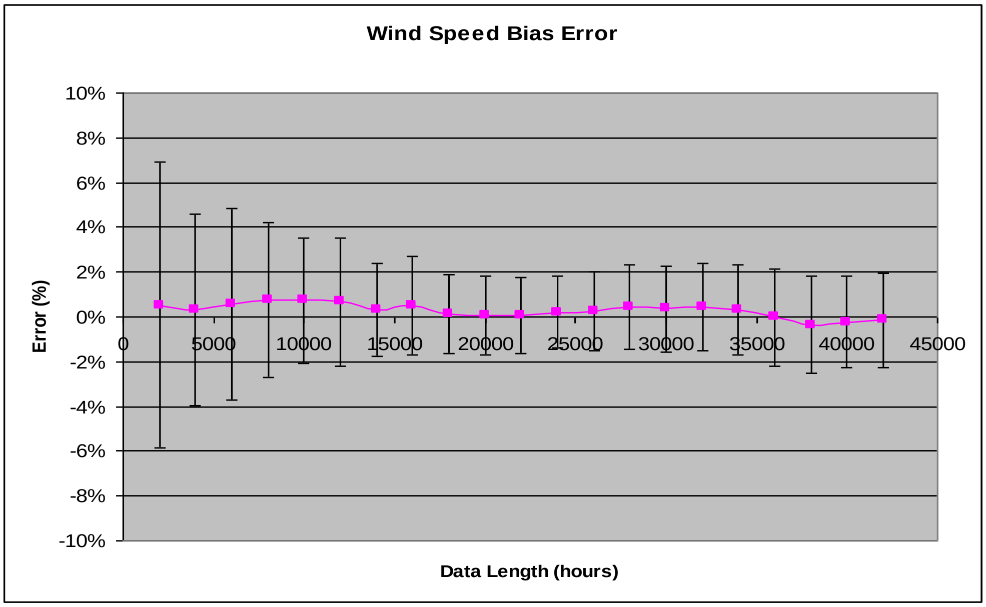


Figure 5.17(a) – Wind speed bias error as a function of concurrent data length for LSQ2 (sliced)

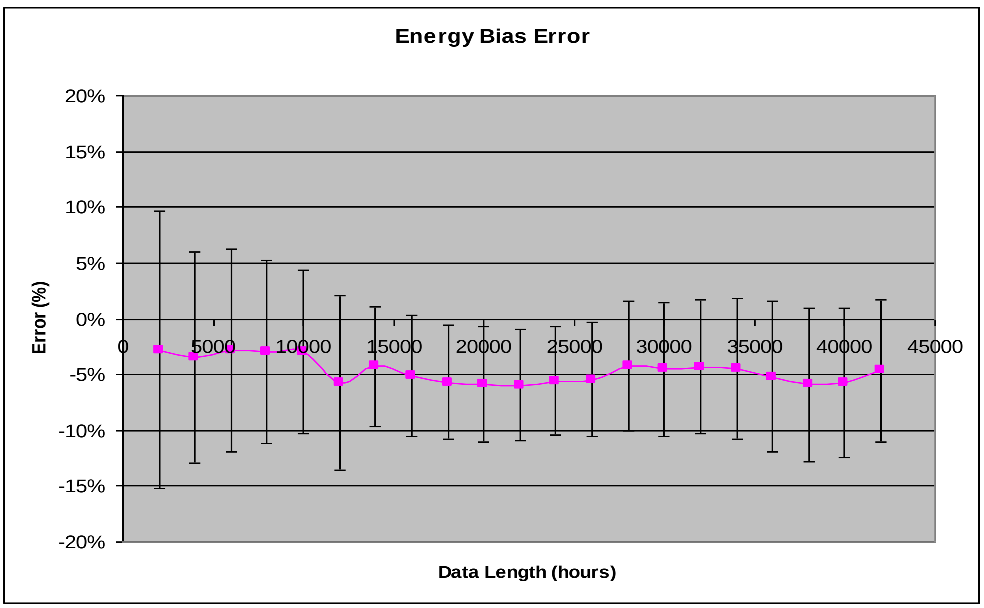


Figure 5.17(b) – Energy bias error as a function of concurrent data length for LSQ2 (sliced)

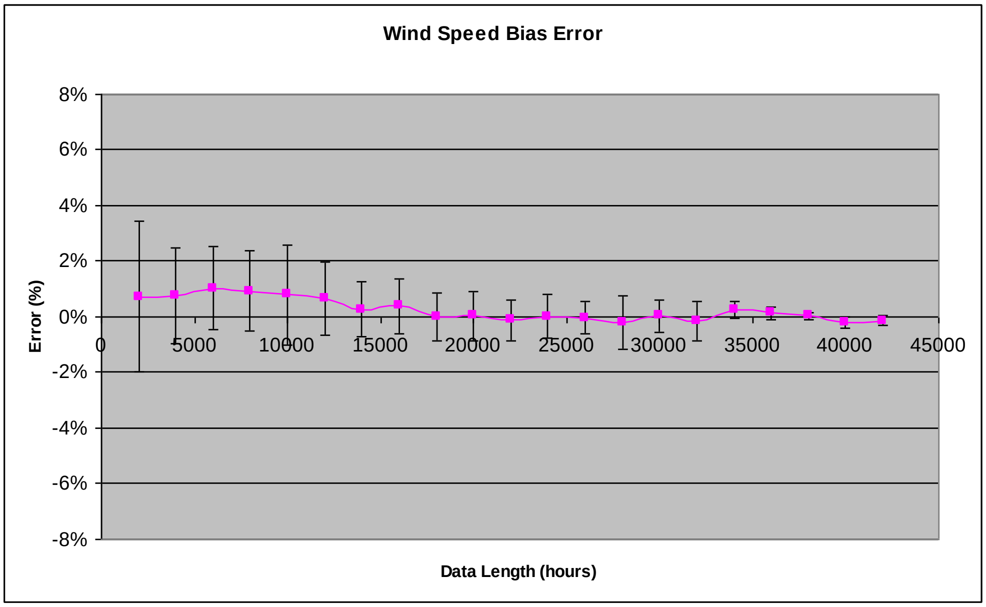


Figure 5.18(a) – Wind speed bias error as a function of concurrent data length for LSQ2 (diced)

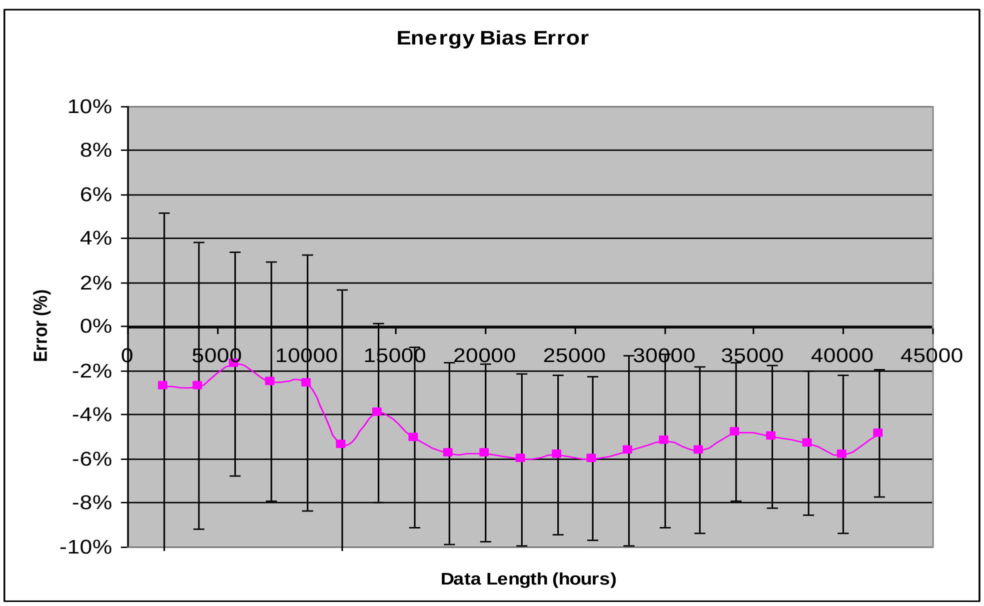


Figure 5.18(b) – Energy bias error as a function of concurrent data length for LSQ2 (diced)

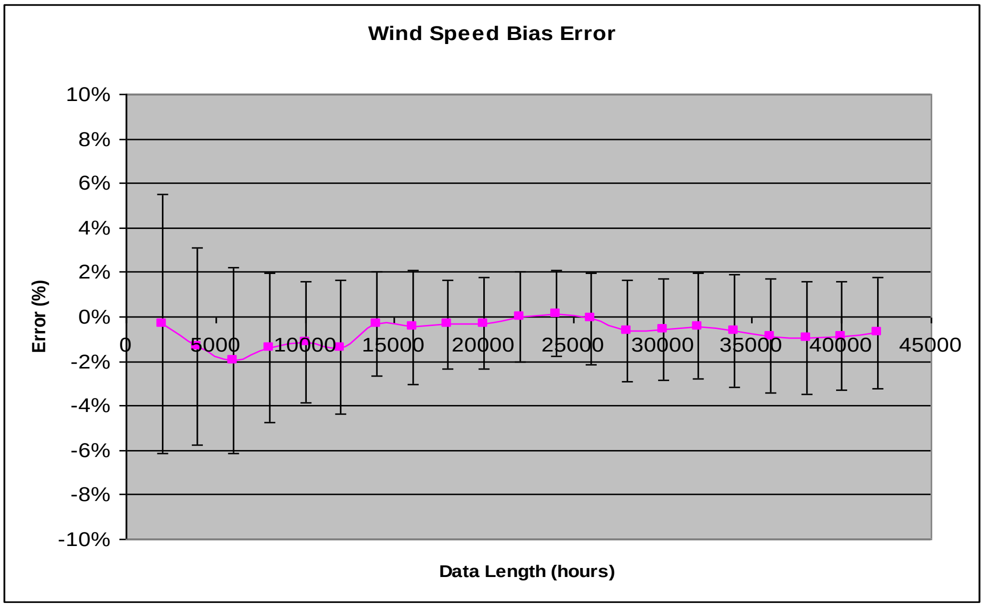


Figure 5.19(a) – Wind speed bias error as a function of concurrent data length for Method B (sliced)

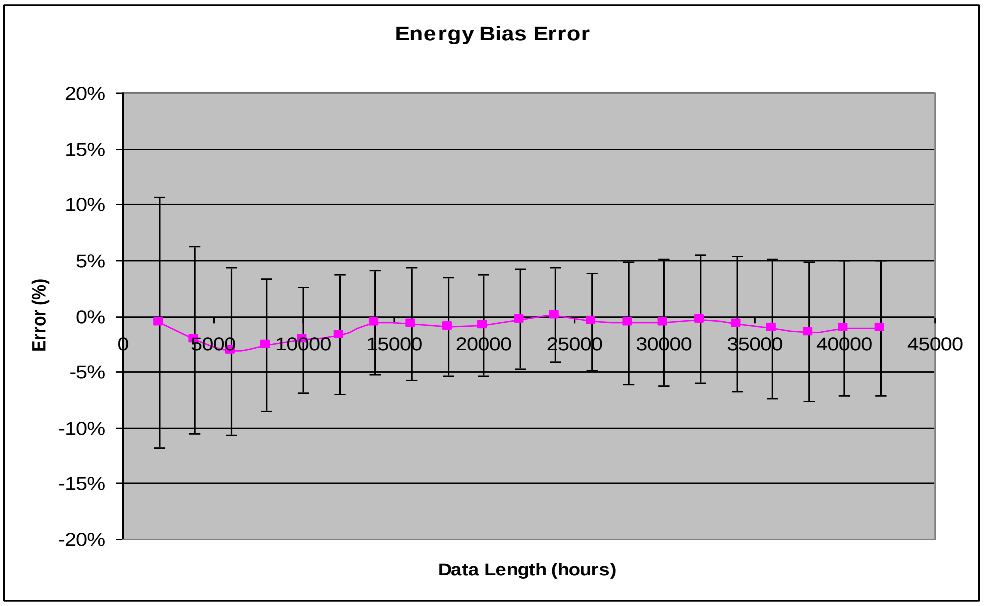


Figure 5.19(b) – Energy bias error as a function of concurrent data length for Method B (sliced)

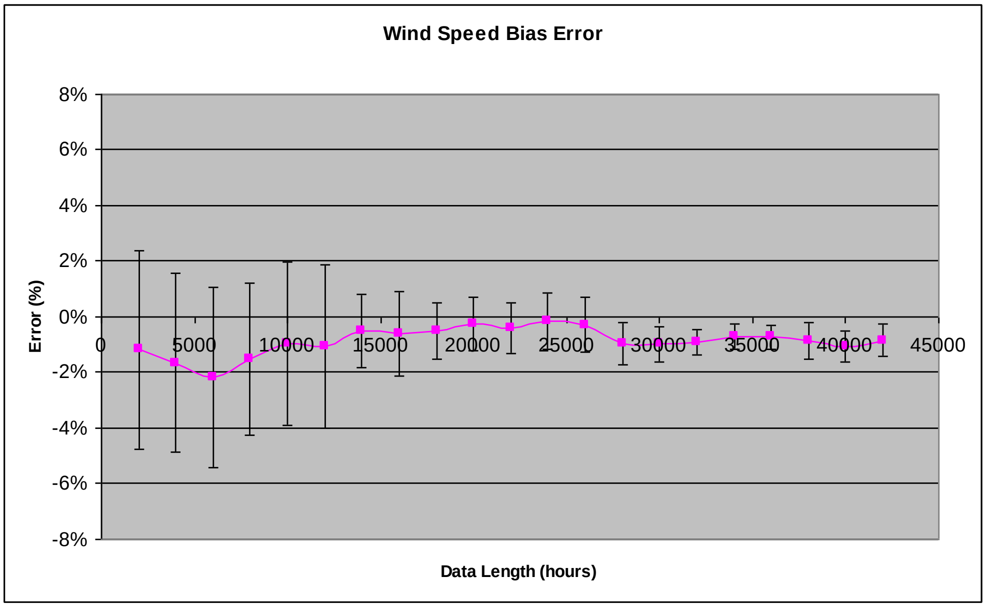


Figure 5.20(a) – Wind speed bias error as a function of concurrent data length for Method B (diced)

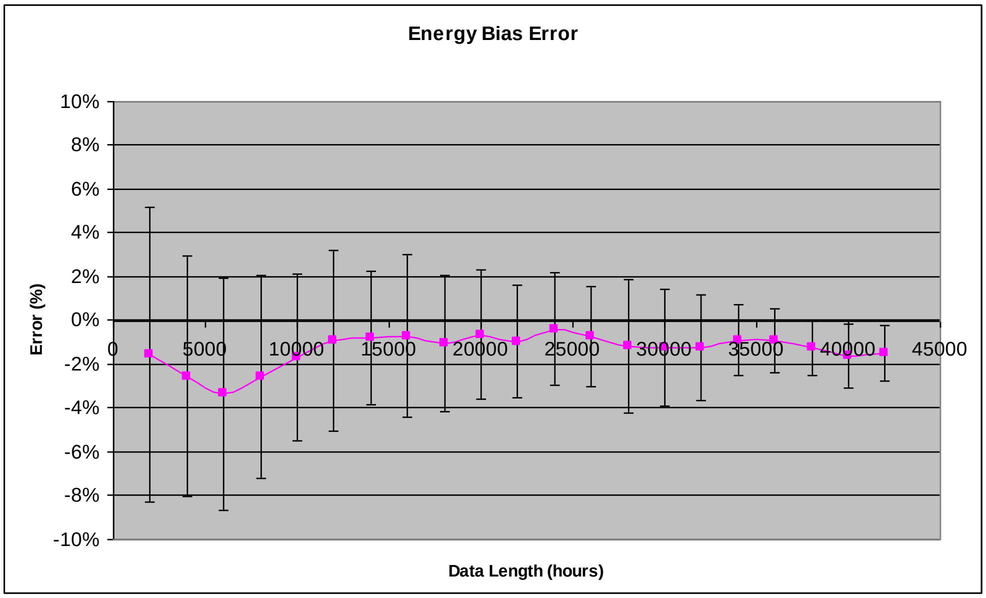


Figure 5.20(b) – Energy bias error as a function of concurrent data length for Method B (diced)

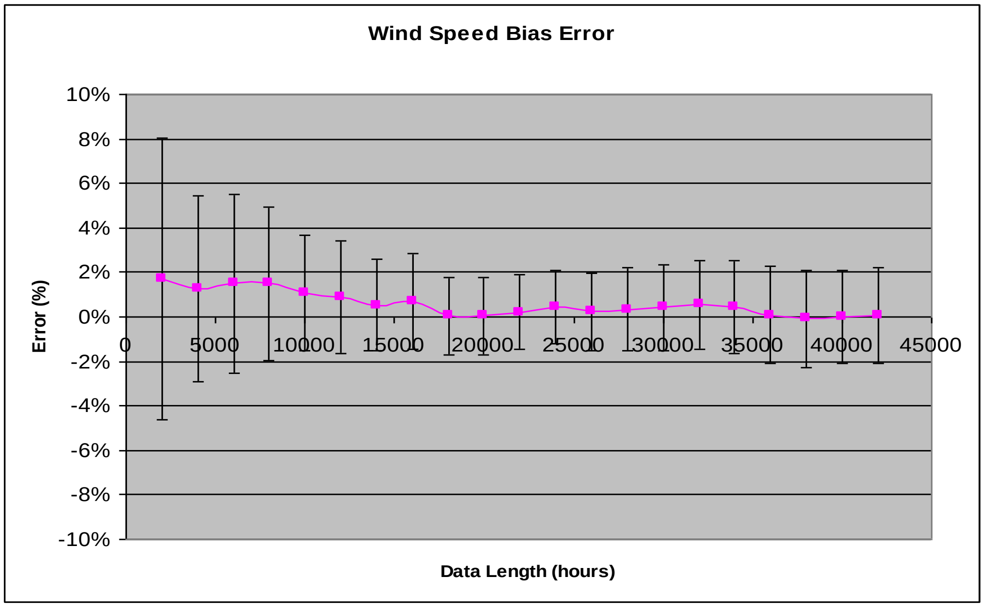


Figure 5.21(a) – Wind speed bias error as a function of concurrent data length for Method A (sliced)

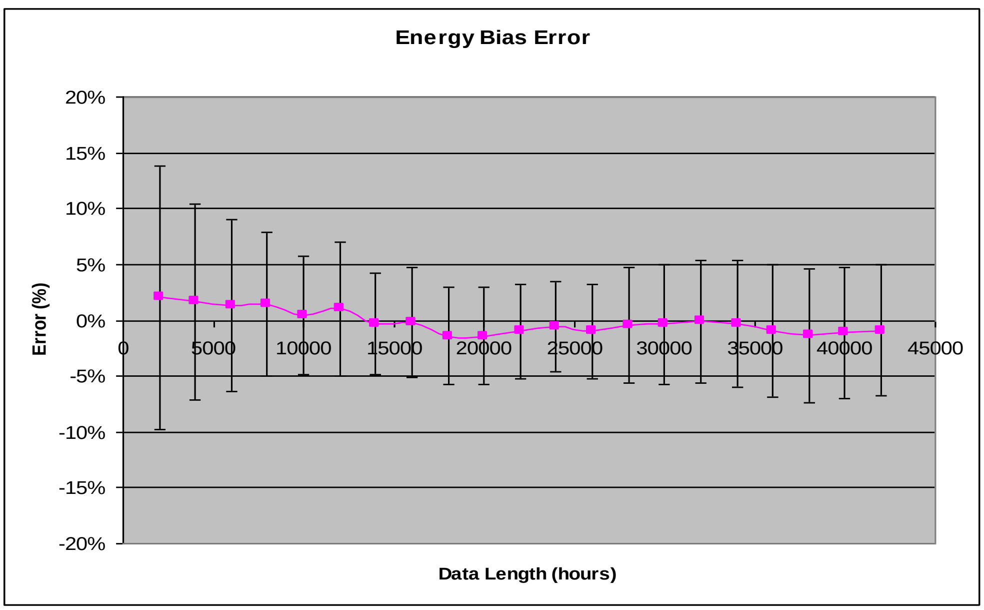


Figure 5.21(b) – Energy bias error as a function of concurrent data length for Method A (sliced)

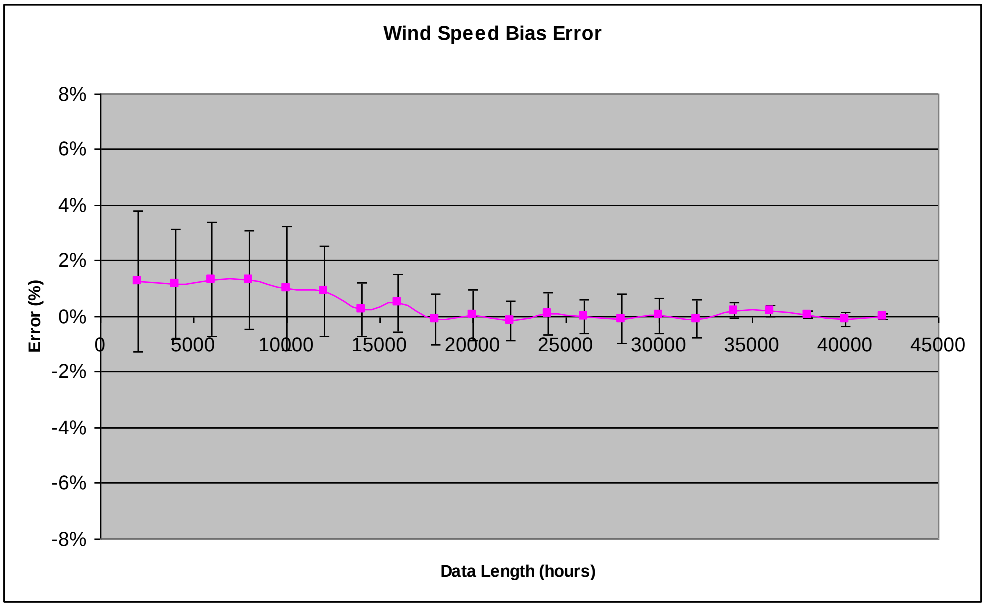


Figure 5.22(a) – Wind speed bias error as a function of concurrent data length for Method A (diced)

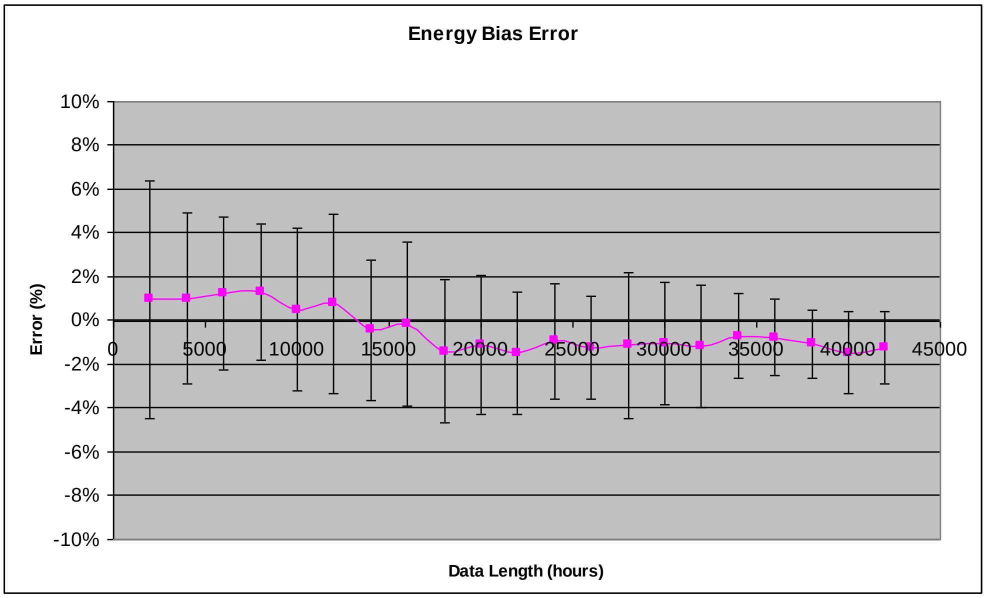


Figure 5.22(b) – Energy bias error as a function of concurrent data length for Method A (diced)

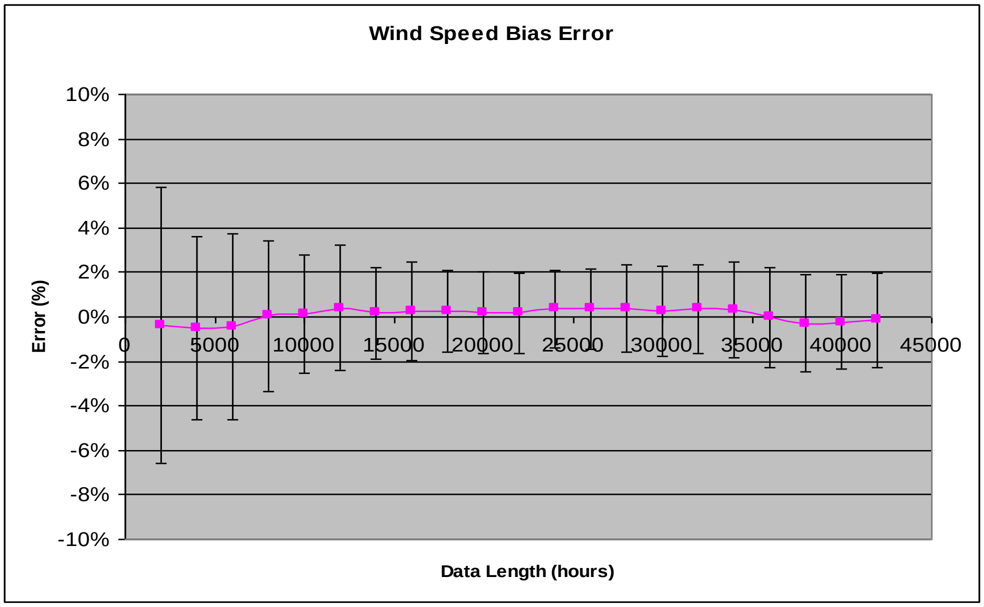


Figure 5.23(a) – Wind speed bias error as a function of concurrent data length for Matrix (sliced)

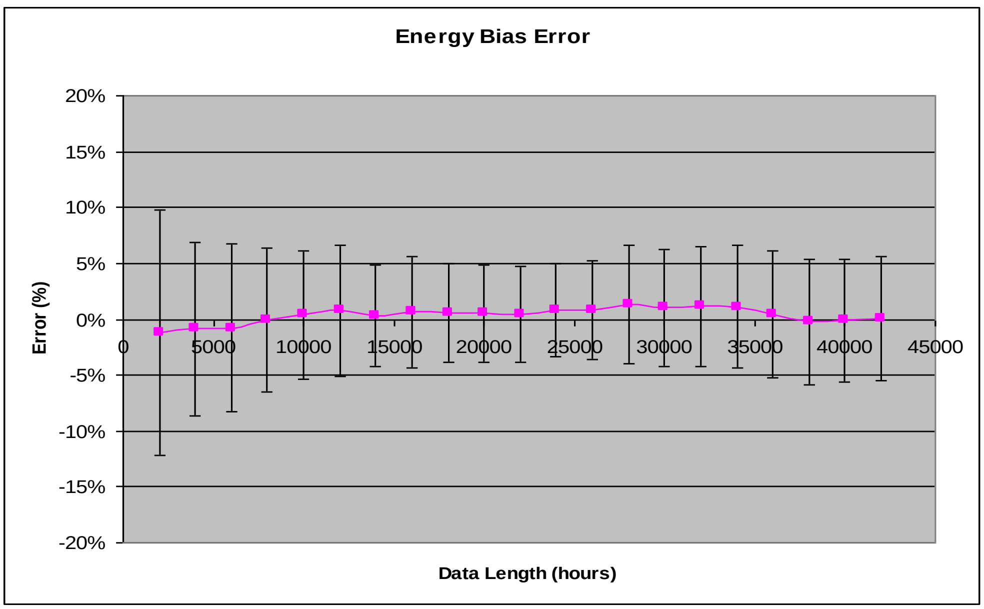


Figure 5.23(b) – Energy bias error as a function of concurrent data length for Matrix (sliced)

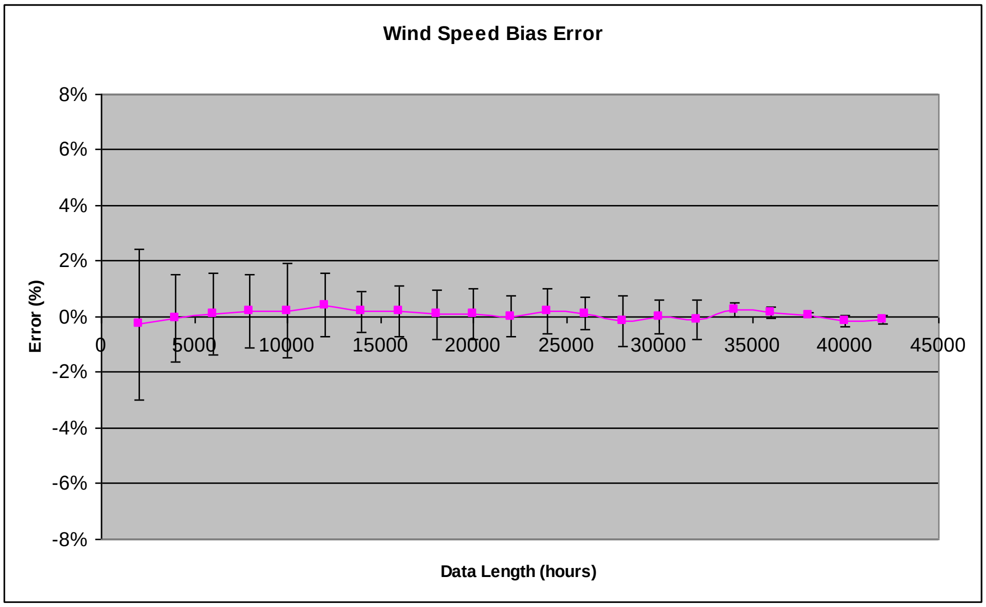


Figure 5.24(a) – Wind speed bias error as a function of concurrent data length for Matrix (diced)

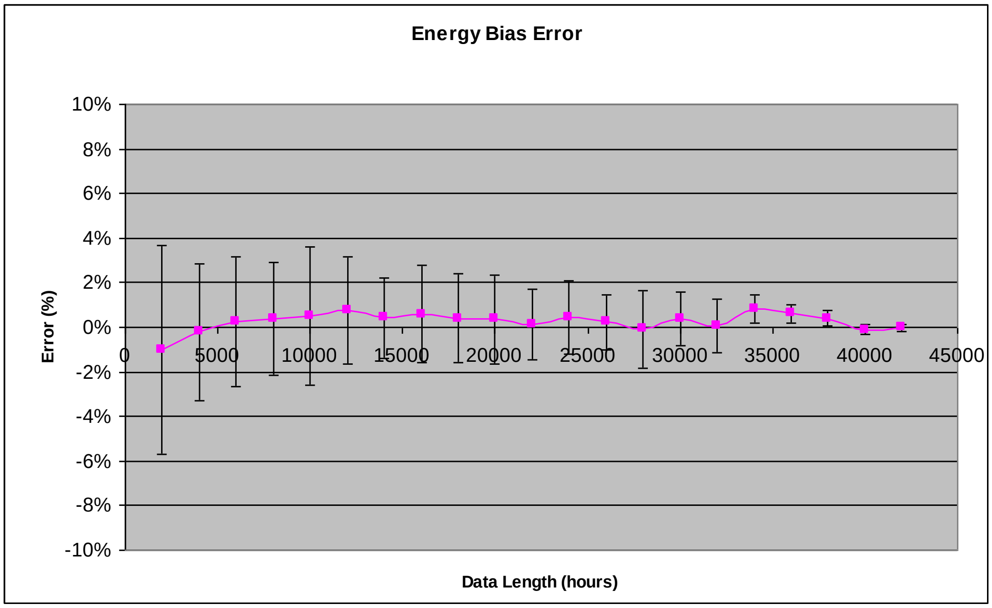


Figure 5.24(b) – Energy bias error as a function of concurrent data length for Matrix (diced)

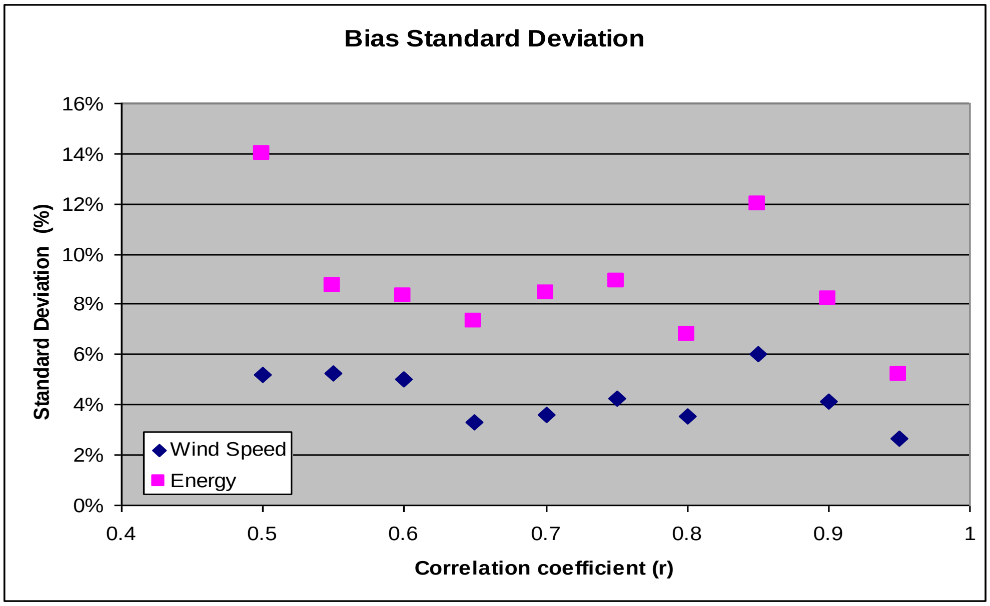


Figure 5.25(a) – Standard deviation of bias error as a function of correlation coefficient for LSQ2 (sliced)

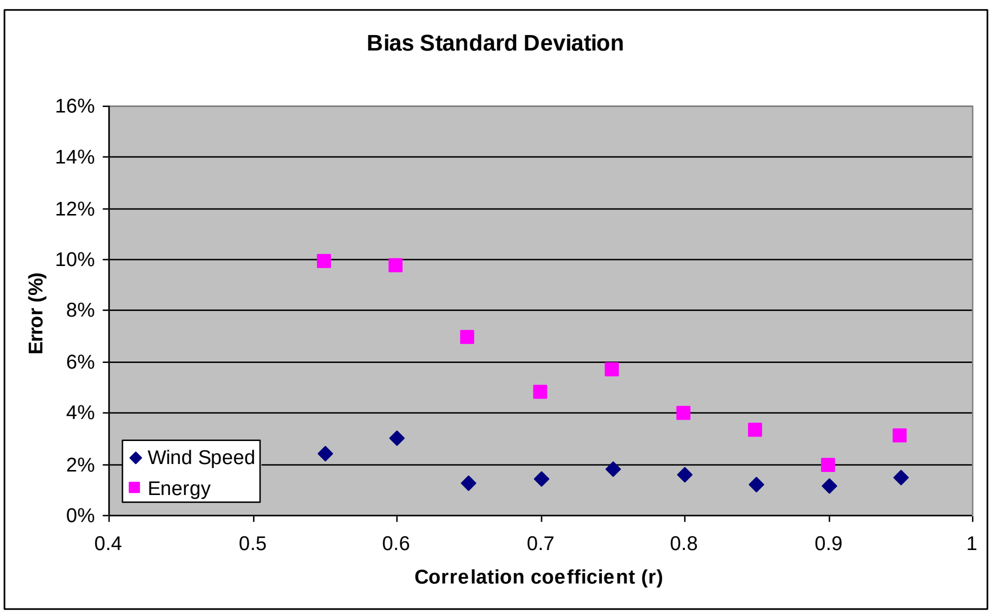


Figure 5.25(b) – Standard deviation of bias error as a function of correlation coefficient for LSQ2 (diced)

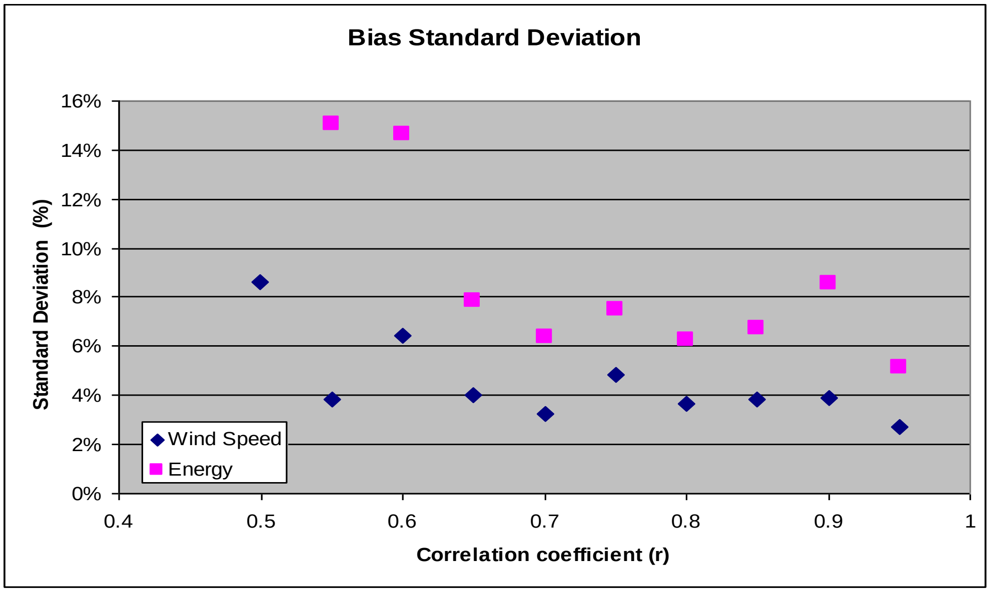


Figure 5.26(a) – Standard deviation of bias error as a function of correlation coefficient for Method B (sliced)

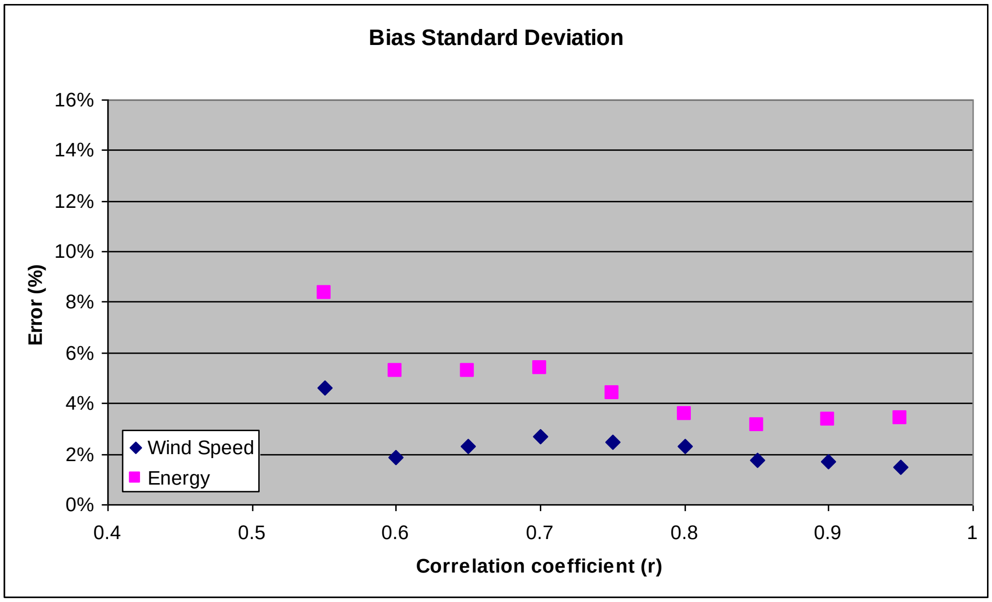


Figure 5.26(b) – Standard deviation of bias error as a function of correlation coefficient for Method B (diced)

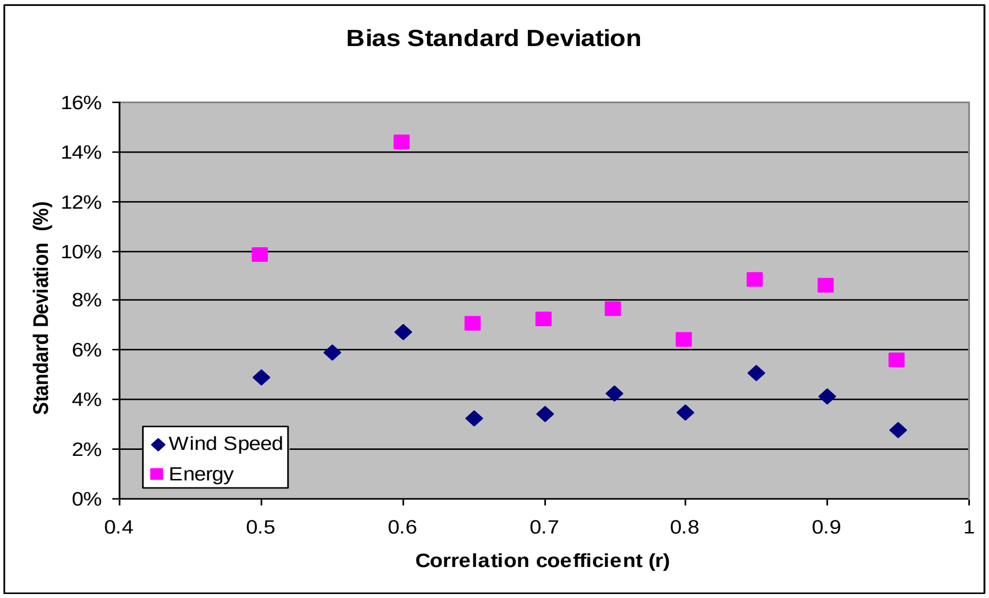


Figure 5.27(a) – Standard deviation of bias error as a function of correlation coefficient for Method A (sliced)

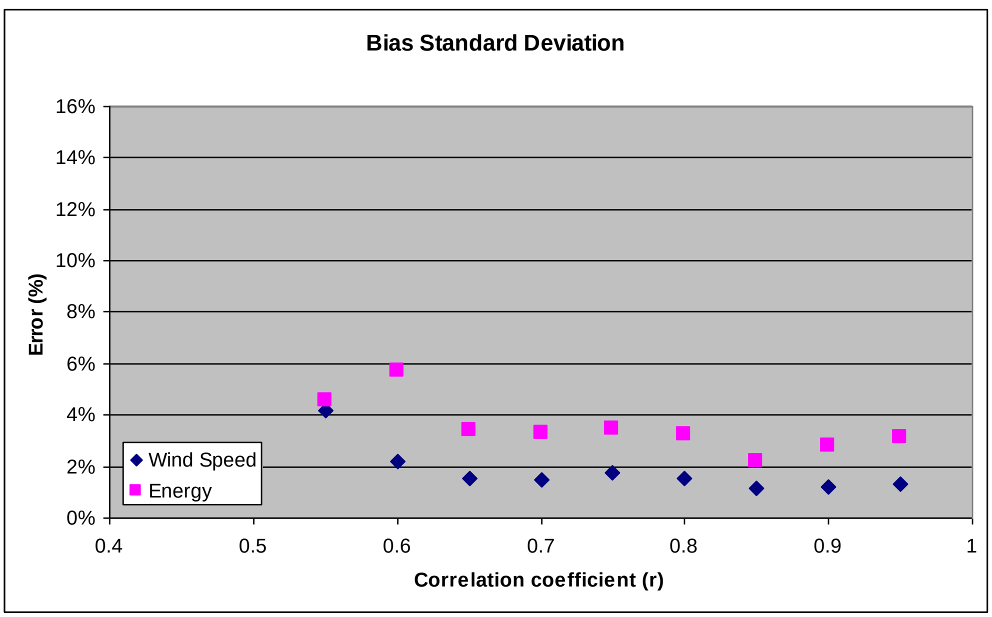


Figure 5.27(b) – Standard deviation of bias error as a function of correlation coefficient for Method A (diced)

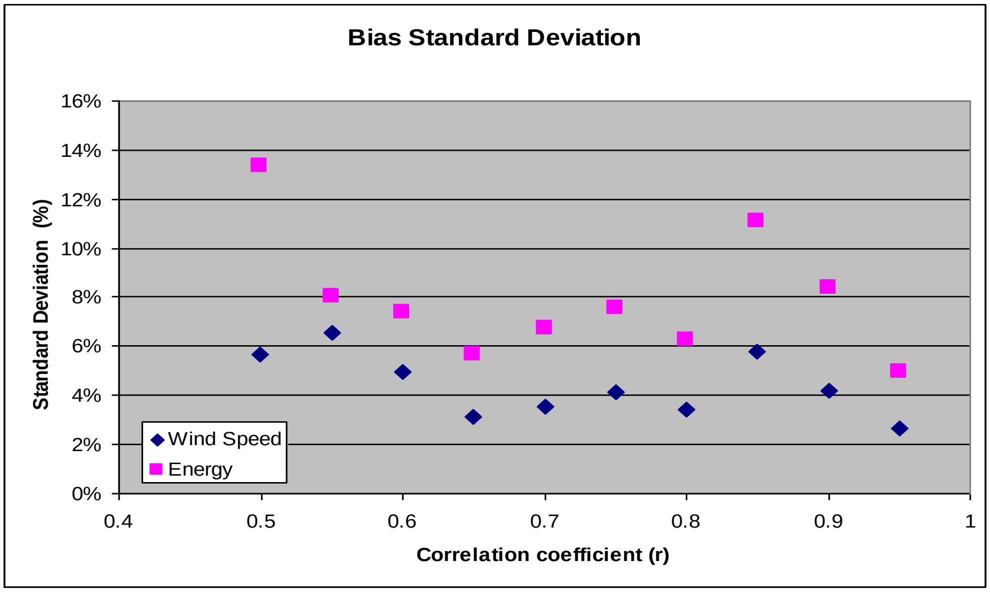


Figure 5.28(a) – Standard deviation of bias error as a function of correlation coefficient for Matrix (sliced)

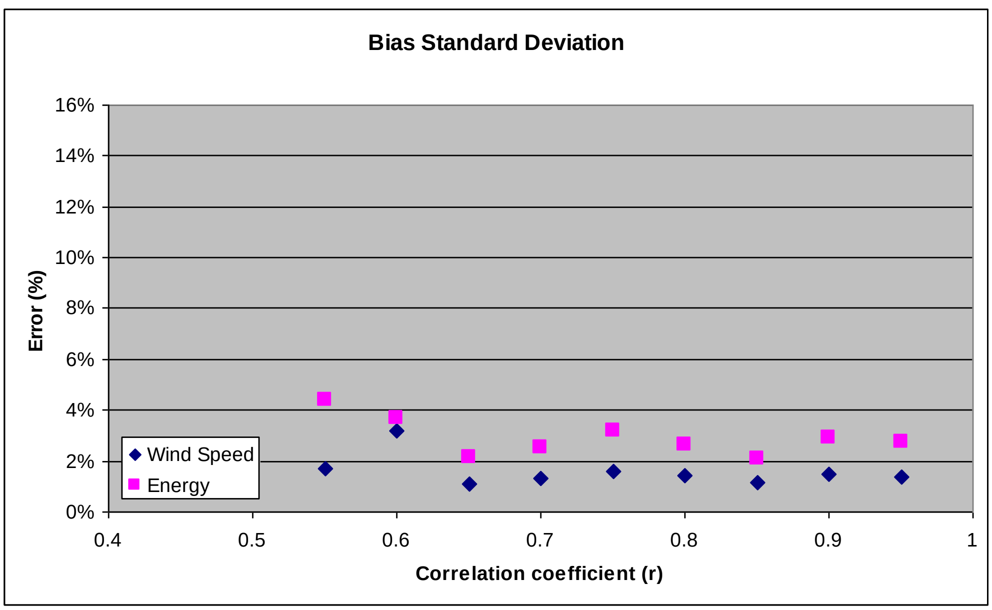


Figure 5.28(b) – Standard deviation of bias error as a function of correlation coefficient for Matrix (diced)